Holostar Thermodynamics

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Abstract

A simple thermodynamic model for the final state of a collapsed, spherically symmetric star is presented. It is assumed that at the end-point of spherically symmetric collapse the particles within the star become ultra relativistic and that their thermodynamic properties can be described by an ideal gas of ultra-relativistic fermions and bosons. The metric at the final stage of collapse is assumed to approach the static metric of the so called holographic solution, a new exact spherically symmetric solution to the Einstein field equations with zero cosmological constant.

If the geometry of a collapsed star can be described by the holostar metric, the established picture of gravitational collapse of a relativistic star changes significantly. The metric-induced "expansion" of space in combination with the quantum mechanical degeneracy pressure of its constituent matter allow a collapsing star to settle down to a thermodynamically stable state, which doesn't have a point-mass at its center, irrespective of the star's mass. The thermodynamical configuration of the holostar is stabilized by a non-zero chemical potential of the relativistic fermions, which is proportional to the local radiation temperature within the holostar. The non-zero chemical potential acts as a natural source for a significant matter-antimatter asymmetry in a self gravitating object.

The final configuration has a radius slightly exceeding the gravitational radius of the star. The radial coordinate difference between gravitational and actual radius is of order of the Planck length and roughly proportional to the square root of the effective number of degrees of freedom of the particles within. The total number of ultra-relativistic particles within the star is proportional to its proper surface-area, measured in units of the Planck-area. The entropy per particle σ can be calculated from the thermodynamic model. $\sigma >\simeq \pi$, nearly independent from the specifics of the thermodynamic model. This is first direct evidence for the microscopic-statistical nature of the Hawking entropy and indicates, that the holographic principle is valid for compact self gravitating objects of any size.

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The free energy within the holostar is minimized to $F = 0$, meaning that - on average - the entropy per particle is equal to the energy per particle per temperature, i.e. $\sigma = \epsilon/T$.

A "Stefan-Boltzmann-type" relation between the local surface temperature and the proper surface area of the star is derived. This relation implies a well defined internal radiation temperature proportional to $1/\sqrt{r}$. The red-shift factor of the holostar's surface with respect to an observer at infinity is proportional to $1/\sqrt{M}$, so that the holostar's temperature at infinity is proportional to $1/M$. The Hawking temperature and -entropy laws are derived from microscopic statistical thermodynamics up to a constant factor.

The factor relating the holostar's temperature at infinity to the Hawking temperature can be expressed in terms of the internal local radiation temperature and the local total matter-density, allowing an experimental determination of this factor. Assuming, that the universe can be described by a large holostar and using the recent experimental data for the CMBR-temperature and the total matter-density the Hawking formula is verified to an accuracy better than 1%.

The surface area of a holostar consists of a two dimensional membrane with high tangential pressure. The membrane's contribution to the entropy and gravitating mass of the holostar is discussed briefly under the assumption, that the membrane consists of a gas of bosons whose number is roughly equal to the number of the interior particles.

A table for the relevant thermodynamic parameters for various combinations of the degrees of freedom and the chemical potentials of the interior particles (fermions and bosons) is given. The total number of particle degrees of freedom at ultra-relativistic energies is estimated to be $f \approx 7000$. The holographic solution preserves the relative ratios of the energy-densities of the fundamental particle species. This allows an estimation of the proton to electron mass-ratio $m_p/m_e \approx f/4$.

The case of a "zero-temperature" holostar and some properties expected from a rotating holostar are discussed briefly.

1   Introduction:

In [6] a variety of new solutions to the field equations of general relativity were derived. These solutions describe a spherically symmetric compact gravitating object with a generally non-zero interior matter-distribution. The boundary of the matter distribution generally lies outside of the object's gravitational radius and consists of a two-dimensional spherical membrane of high surface-tension/pressure, whose energy-content is comparable to the gravitating mass of the object.

Some of the new solutions don't contain a point-singularity at the origin, indicating that spherically symmetric collapse of a large star or a galactic core might not necessarily end up in a black hole of the Schwarzschild (vacuum) type, if the pressure becomes anisotropic.

The new solutions cover a wide range of possibilities. The most promising solutions appear to be those characterized by a mass-density of $\rho \propto 1/(8\pi r^2)$

within the interior.

The solution with $\rho = 1/(8\pi r^2)$, the so called holographic solution, or short "holostar", is of particular interest. Its geometric properties have been discussed in [7] in some detail. The holostar is characterized by the property, that the stress-energy content of the holostar's membrane is equal to its gravitating mass. Alternatively the holostar's total gravitating mass can be derived by an integral over the *trace* of the stress-energy tensor, a Lorentz invariant quantity.

The interior matter-state of the holographic solution can be interpreted as a collection of radially outlayed strings, with the end-points of the strings attached to the holostar's boundary membrane. Every string segment attached to the membrane occupies a membrane segment of exactly one Planck area. The interior strings are densely packed, their mutual transverse separation is exactly one Planck area. This dense package might be at the heart of the explanation, why the holostar doesn't collapse to a singularity, regardless of it's size. For a more detailed discussion of the string nature of the holographic solution see [8].

Although the holographic solution has a strong string character, its interior matter state can also be interpreted in terms of particles. In this paper a simple thermodynamic model for the interior matter state is explored, which provides a genuine microscopic statistical explanation for the Hawking temperature and entropy of a compact self gravitating object and gives a thermodynamic explanation for the matter-antimatter asymmetry in curved space-time.

2 A short introduction to the holographic solution

The holographic solution is an exact solution to the Einstein field equations with zero cosmological constant[1]. The spherically symmetric metric of the holographic solution has been derived in [6] :

$$ds^2 = g_{tt}(r)dt^2 - g_{rr}(r)dr^2 - r^2 d\Omega^2 \tag{1}$$

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[1]The holographic solution can be viewed as the simplest possible solution to the field equations. Although it contains matter, it is much simpler than a pure vacuum solution. How can this be? General relativity is a *non-linear* theory. Even its *vacuum* equations are non-linear and all of its known solutions (=classical black holes) quite complicated. Therefore most of general relativity's practitioners tend to think that *all* relevant solutions to the field equations *must* be non-linear and highly complex. There is a general feeling that the solutions will become even more complicated when matter is introduced into the theory. Linearizing the field equations is viewed as a mere approximation, appropriate only in the weak field limit. The full result should always be non-linear. But this is not the case: The introduction of *string type* matter into the general theory of relativity *simplifies* the field equations in a very essential way. In the spherically symmetric case it is very easy to see that the field equations are *linearized*, if the matter follows a string equation of state $\rho = -P_r$. That string type matter *must* reduce the complexity of the field equations becomes clear, if one realizes that the active gravitational mass-density of a string is always *zero*. Neither the vacua of the classical black hole solutions nor other types of matter have this property. But the active gravitational mass density can be viewed as the "true" source of the gravitational field. The equations for the local proper geodesic acceleration in a local Minkowski frame have the active gravitational mass-density as a source term.

$$g_{tt}(r) = 1/g_{rr}(r) = \frac{r_0}{r}(1 - \theta(r - r_h)) + (1 - \frac{r_+}{r})\theta(r - r_h) \tag{2}$$

with

$$r_h = r_+ + r_0$$

$$r_+ = 2M$$

All quantities are expressed in geometric units $c = G = 1$. For clarity $\hbar$ will be shown explicitly. θ and δ are the Heavyside-step functional and the Dirac-delta functional respectively. r_h denotes the radial coordinate position of the holostar's surface, which divides the space-time manifold into an interior source region with a non-zero matter-distribution and an exterior vacuum space-time. r_+ is the radial coordinate position of the gravitational radius (Schwarzschild radius) of the holostar. r_+ is directly proportional to the gravitating mass $M = r_+/2$. r_0 is a fundamental length parameter.[2]

The matter fields (mass density, principal pressures) of any spherically symmetric gravitationally bound object can be derived from the metric by simple differentiation (see for example [6]). For the discussion in this paper only the *radial* metric coefficient $g_{rr}(r)$ is essential. In the spherically symmetric case the total mass-energy density ρ can be calculated *solely* from the radial metric coefficient.[3] For any spherically symmetric self gravitating object the following general relation holds:

$$\left(\frac{r}{g_{rr}}\right)' = 1 - 8\pi r^2 \rho \tag{3}$$

It is obvious from the above equation, that a matter-density $\rho = 1/(8\pi r^2)$ is special. It renders the differential equation for g_{rr} homogeneous and leads to a strictly linear dependence between g_{rr} and the radial distance coordinate r.

With g_{rr} given by equation (2) the energy-density turns out to be:

$$\rho(r) = \frac{1}{8\pi r^2}(1 - \theta(r - r_h)) \tag{4}$$

Within the holostar's interior the mass-energy density follows an inverse square law. Outside of the membrane, i.e. for $r > r_h$, it is identical zero. Note, that r_h must not necessarily be finite.

In the following discussion the argument $(r - r_h)$ of the θ- and δ-distributions will be omitted.

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[2] r_0 has been assumed to be roughly twice the Planck-length in [6, 7]. The analysis in [5] indicates $r_0^2 \simeq 4\sqrt{3/4}$ at low energies. In this paper a more definite relationship in terms of the total number of particle degrees of freedom at high temperatures will be derived.

[3] If the principal pressures are to be derived from the metric, one must also know the *time-coefficient* of the metric. This can be loosely interpreted such, that pressure is a "time-dependent" phenomena, i.e. the physical origin of pressure is intimately related to the (un-ordered) motion of the particles, whereas the mass-energy density of a static body not necessarily needs the concept of motion for it's definition.

The radial and tangential pressures also follow from the metric:

$$P_r = -\rho = -\frac{1}{8\pi r^2}(1 - \theta) \tag{5}$$

$$P_\theta = P_\varphi = \frac{1}{16\pi r_h}\delta \tag{6}$$

P_r is the radial pressure. It is equal in magnitude but opposite in sign to the mass-density. P_θ denotes the tangential pressure, which is zero everywhere, except for a δ-functional at the holostar's surface. The "stress-energy-content" of the two principal tangential pressure components in the membrane is equal to the gravitating mass M of the holostar.

In order to determine the principal pressures from the metric, the time-coefficient of the metric g_{tt} must be known. For the holostar equation of state with $P_r = -\rho$ we have $g_{tt} = 1/g_{rr}$. Other equations of state lead to different time-coefficients, and therefore different principal pressures.

Neither the particular form of the time-coefficient of the metric, nor the particular form of the principal pressures are important for the main results derived in this paper, which are based on equilibrium thermodynamics, where time evolution is irrelevant.[4] The essential assumptions are:

- spherical symmetry

- a radial metric coefficient $g_{rr} = r/r_0$

- a total energy density $\rho = 1/(8\pi r^2)$

- microscopic statistical thermodynamics of an ideal gas of ultra-relativistic fermions and bosons (in the context of the grand-canonical ensemble)

If the validity of Einstein's field equations with zero cosmological constant is assumed, conditions two and three are interchangeable.

In the following sections I assume that r_0^2 is nearly constant, i.e. more or less independent of the size of the holostar and comparable to the Planck area $A_{Pl} = \hbar$:

$$r_0{}^2 = \beta r_{Pl}{}^2 = \beta\hbar \tag{7}$$

This assumption will be justified later.

3 A simple derivation of the Hawking temperature and entropy

The interior metric of the holostar solution is well behaved and the interior matter-density is non-zero. The solution is static: The matter appears to exert a radial pressure preventing further collapse to a point singularity. However,

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[4]As long as the relevant time scale is long enough, that thermal equilibrium can be attained.

the solution gives no direct indication with respect to the state of the interior matter and the origin of the pressure.

In this section I will discuss a very simple model for the interior matter state of the holostar, which is able to explain many phenomena attributed to black holes. Let us assume that the interior matter distribution is dominated by ultra-relativistic weakly interacting fermions and the pressure is produced by the exclusion principle. Due to spherical symmetry the mean momentum of the fermions $p(r)$ and their number density per proper volume $fn(r)$ will only depend on the radial distance coordinate r. f denotes the effective number of degrees of freedom of the fermions. For ultra-relativistic fermions the local energy-density will be given by the product of the number density of the fermions and their mean momentum. This energy density must be equal to the interior mass-energy density of the holostar:

$$\rho = p(r)fn(r) = \frac{1}{8\pi r^2} \tag{8}$$

If the fermions interact only weakly, their mean momenta can be estimated by the exclusion principle:

$$p(r)^3 \frac{1}{n(r)} = (2\pi\hbar)^3 \tag{9}$$

These two equations can be solved for $p(r)$ and $n(r)$:

$$p(r) = \frac{\hbar^{\frac{3}{4}}\pi^{\frac{1}{2}}}{f^{\frac{1}{4}}}\frac{1}{r^{\frac{1}{2}}} \tag{10}$$

$$fn(r) = \frac{f^{\frac{1}{4}}}{\hbar^{\frac{3}{4}}8\pi^{\frac{3}{2}}}\frac{1}{r^{\frac{3}{2}}} \tag{11}$$

The mean momenta of the fermions within the holostar fall off from the center as $1/r^{1/2}$ and the number density per proper volume with $1/r^{3/2}$. Similar dependencies, however without definite factors, have already been found in [7] by analyzing the geodesic motion of the interior massless particles in the holostar-metric. It is remarkable, that equilibrium thermodynamics combined with the uncertainty principle gives the same results as the geodesic equations of motion. This is not altogether unexpected. In [4] it has been shown, that the field equations of general relativity follow from thermodynamics and the Bekenstein entropy bound [2].

The momentum of the fermions at a Planck-distance $r = r_{Pl} = \sqrt{\hbar}$ from the center of the holostar is of the order of the Planck-energy $E_{Pl} = \sqrt{\hbar}$. It is also interesting to note, that for both quantities $p(r)$ and $n(r)$ the number of degrees of freedom f can be absorbed in the radial coordinate value $r \rightarrow \sqrt{f}\,r$, so that p and n effectively depend on $\sqrt{f}\,r$. We will see later that the square root of f plays an important role in the scaling of the fundamental length parameter r_0.[5]

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[5]Perhaps it would be better to reformulate the above statement, by saying that the fundamental area $4\pi r_0^2$ scales with f.

From (10) one can derive the following momentum-area law for holostars, which resembles the Stefan-Boltzmann law for radiation from a black body:

$$p(r)^4 r^2 f = \hbar^3 \pi^2 \tag{12}$$

Note that this law not only refers to the holostar's surface $(r = r_h)$ but is valid for any concentric spherical surface of radius r within the holostar. Therefore it is reasonable to assume that the holostar has a well defined interior temperature $T(r)$ proportional to the mean momentum $p(r)$:

$$p(r) = \sigma T(r) \tag{13}$$

σ is a constant factor. We will see later, that it is related to the entropy per particle.

The local surface temperature of the holostar is given by:

$$T(r_h) = \frac{p(r_h)}{\sigma} = \frac{\hbar^{\frac{3}{4}} \pi^{\frac{1}{2}}}{\sigma f^{\frac{1}{4}}} \frac{1}{\sqrt{r_h}} \tag{14}$$

The surface redshift z is given by:

$$z = \frac{1}{\sqrt{g_{tt}(r_h)}} = \sqrt{g_{rr}(r_h)} = \sqrt{\frac{r_h}{r_0}} \tag{15}$$

where $g_{tt}(\infty) = 1$ is assumed.

The local surface temperature can be compared to the Hawking temperature of a black hole. The Hawking temperature is measured at infinity. Therefore the red-shift of the radiation emitted from the holostar's surface with respect to an observer at spatial infinity has to be taken into account, by dividing the local temperature at the surface by the gravitational red shift factor z. With $g_{rr}(r_h) = r_h{}^{1/2}(\beta\hbar)^{-1/4}$ we find:

$$T_\infty = \frac{T(r_h)}{\sqrt{g_{rr}(r_h)}} = \frac{\pi^{\frac{1}{2}}}{\sigma} \left(\frac{\beta}{f}\right)^{\frac{1}{4}} \frac{\hbar}{r_h} \tag{16}$$

The surface-temperature measured at infinity has the same dependence on the gravitational radius r_h as the Hawking temperature, which is given by:

$$T_H = \frac{\hbar}{4\pi r_h} = \frac{\hbar}{8\pi M} \tag{17}$$

We get the remarkable result, that - up to a possibly different constant factor - the Hawking temperature of a spherically symmetric black hole and the respective temperature of the holostar at infinity are equal.

As the Hawking temperature of a black hole only depends on the properties of the exterior space-time, and the exterior space-times of a black hole and the holostar are equal (up to a small Planck-sized region outside the horizon), it is reasonable to assume that the Hawking temperature should be the true temperature of a holostar measured at spatial infinity. With this assumption,

the constant σ can be determined by setting the temperatures of equations (16) and (17) equal:

$$\sigma = \left(\frac{\beta}{f}\right)^{\frac{1}{4}} 4\pi^{\frac{3}{2}} \tag{18}$$

The total number of fermions within the holostar is given by the proper integral over the number-density:

$$N = \int fn(r)dV \tag{19}$$

dV is the proper volume element, which can be read off from the metric:

$$dV = 4\pi r^2 \sqrt{g_{rr}}dr = 4\pi r^{\frac{5}{2}} \left(\beta\hbar\right)^{-\frac{1}{4}} dr \tag{20}$$

Integration over the total interior volume of the holostar gives:

$$N = \left(\frac{f}{\beta}\right)^{\frac{1}{4}} \frac{1}{4\pi^{\frac{3}{2}}} \frac{\pi r_h^2}{\hbar} = \frac{1}{\sigma} \frac{A}{4\hbar} = \frac{S_{BH}}{\sigma} \tag{21}$$

S_{BH} is the Bekenstein-Hawking entropy for a spherically symmetric black hole with horizon surface area A.

Therefore the number of fermions within the holostar is proportional to its surface area and thus proportional to the Hawking entropy. This result is very much in agreement with the holographic principle, giving it quite a new and radical interpretation: The degrees of freedom of a highly relativistic self-gravitating object don't only "live on the surface", the object contains a definite number of particles[6] and their total number is proportional to the object's surface area, measured in units of the Planck-area, $A_{Pl} = \hbar$. This result is an immediate consequence of the interior metric $g_{rr} \propto r$, the energy-momentum relation for relativistic particles $E = p$ and the exclusion principle. It can be easily shown, that for any other spherically symmetric metric, for example $g_{rr} \propto r^n$, the number of interior (fermionic) particles is not proportional to the boundary area.[7]

From equation (21) we can see that σ is the entropy per particle. This allows a rough estimate of β: The entropy of an ultra-relativistic particle should be of order unity ($\sigma \approx 3 - 4$). If we count the degrees of freedom of all fermions in the Standard Model of particle physics (three generations of quarks and leptons including the spin, color and antiparticle degrees of freedom), their combined

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[6]This is not true for the Einstein-Maxwell vacuum black hole solutions with event horizon. Due to the nature of the event horizon and its accompanying singularity the number and nature of the particles within a black hole - or rather gone into the black hole - is indefinite.

[7]Quite interestingly, the $N \propto r_h^2$ law can be derived quite similar to the derivation of the Chandrasekhar-limit for a white dwarf star, by assuming that the sum of "gravitational energy" $E_{grav} \propto M/r$ and kinetic energy $E_{kin} \propto Np$ has an extremum (in fact, a maximum!). However, for the determination of the "gravitational energy" of the star the proper radius $r_p = r \left(r/r_0\right)^{1/2}$ must be used instead of the radial coordinate r.

number is 90. The number of bosonic degrees of freedom (8 gluons, 4 electro-weak particles) is 24, disregarding the graviton and assuming the W- and Z-bosons to be massless (above the energy of the electro-weak phase transition). With the usual counting rule[8], weighting the fermionic degrees of freedom with 7/8, one gets $f = 102.75$. Supersymmetry essentially doubles this number. It is expected, that a unified theory will not vastly exceed this number. For $\sigma = 3$ and $f = 256$ we find $4\pi\beta \simeq 1.06$. This justifies the assumption, that the fundamental length parameter r_0 should be roughly equal to the Planck-length.

By help of equation (18) the local temperature can be expressed in terms of β alone:

$$T(r) = \frac{\hbar^{\frac{3}{4}}}{4\pi\beta^{\frac{1}{4}}} \frac{1}{r^{\frac{1}{2}}} = \frac{1}{4\pi} \frac{\hbar}{(r_0 r)^{\frac{1}{2}}} \tag{22}$$

Note that β depends explicitly on the (effective) number of degrees of freedom f of the ultra-relativistic particles within the holostar via equation (18). At the center of the holostar all the fermion momenta are comparable to the Planck energy, as can be seen from equation (22). All fermions of the Standard Model of particle physics will be ultra-relativistic. Quite likely there will be other fundamental particles of a grand unified theory (GUT), as well as other entities such as strings and branes. Thus, close to the holostar's center the number of ultra-relativistic degrees of freedom will be at its maximum and β will be close to unity. The farther one is distanced from the center, the lower the local temperature gets. At $r \approx 10^6 km$ the electrons will become non-relativistic.[9] The only particles of the Standard Model that remain relativistic at larger radial positions will be the neutrinos. If all neutrinos are massive, the mass of the lightest neutrino will define a characteristic radius of the holostar, beyond which there are no relativistic fermions contributing to the holostar's internal pressure. If at least one of the neutrinos is massless, there will be no limit to the spatial extension of a holostar.

Note, that the radial coordinate position at which the holostar's interior radiation temperature is equal to the temperature of the cosmic microwave background radiation, $T_{CMBR} = 2.725\,K$, corresponds to roughly $r \approx 10^{28} m \approx 10^{12}\,ly$, i.e. quite close to the radius of the observable universe. This is just one of several coincidences, which point to the very real possibility, that the holostar or a variant thereof actually might serve as an alternative, beautifully simple model for the universe. For a more detailed discussion including some definite cosmological predictions, which are all experimentally verified within an error of maximally 15 % see [7].

Whenever the temperature within the holostar becomes comparable to the mass of a particular fermion species, a phase transition is expected to take place at the respective r-position. Such a transition will lower the effective value of

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[8]This rule is only true, when the chemical potential of fermions and bosons is zero.

[9]Note that r is not equal to the proper distance to the center. The proper distance scales with $r\sqrt{r/r_0}$. At $r \approx 10^9 m$ the proper distance to the center is roughly $10^{30} m$, i.e. already vastly exceeding the current (Hubble-) radius of the observable universe.

f, as one of the particles "freezes" out.[10] Whenever f changes, either σ or β must adjust due to equation (18). The question is, whether σ or β (or both) will change. Presumably σ will at least approximately retain a constant value: The entropy per ultra-relativistic fermion, as well as the mean particle momentum per temperature, appears to be a local property which should not depend on the (effective) number of degrees of freedom of the particles at a particular r-position.

Under the assumption that σ is nearly constant, the ratio of β/f must be nearly constant as well, as can be seen from equation (18). Whenever f changes, β will adjust accordingly. Lowering the effective number of degrees of freedom leads to a flattening of the temperature-curve, as heat (and entropy) is transferred to the remaining ultra-relativistic particles. At any radial position of a phase transition, where a fermion becomes non-relativistic and annihilates with its anti-particle, the temperature is expected to deviate from the expression $T \propto 1/\sqrt{r}$. This is quite similar to what is believed to have happened in the very early universe, when the temperature fell below the electron-mass threshold and the subsequent annihilation of electron/positron pairs heated up the photon gas, keeping the temperature of the expanding universe nearly constant until all positrons were destroyed.

If the "freeze-out" happens without significant heat and entropy transfer to the remaining gas of ultra-relativistic particles, such as when the particle that "freezes" out has an appreciable non-zero chemical potential, the effective value of f will remain nearly constant, which would imply that β be nearly constant as well. In this case β as well as f would be nearly constant universal quantities. There is evidence that this might actually be the case.[11]

4   Thermodynamics of an ultra-relativistic fermion and boson gas

In this section I will discuss a somewhat more sophisticated model for the thermodynamic properties of the holostar.

As has been demonstrated in the previous section, if the holostar contains at least one fermionic species, its properties very much resemble the Schwarzschild vacuum black hole solution, when viewed from the outside: Due to Birkhoff's theorem the external gravitational field cannot be distinguished from that of a Schwarzschild black hole. Its temperature measured at infinity is proportional to the Hawking temperature.

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[10]The effective value of f must not necessarily change much. If we have a matter-antimatter asymmetry and the chemical potential of the fermionic species that "freezes out" is non zero (and higher than the temperature), there is a chance that the effective value of f remains nearly constant: A significant part of the energy density of the frozen out degrees of freedom will "survive" in the fermion with the high chemical potential (a non-zero chemical potential "prefers" matter over antimatter - or vice versa - and thus forbids the complete annihilation of the fermionic species that is becoming non-relativistic).

[11]See the discussion in section 4 and the related discussions in [5, 7].

Due to its non-zero surface-temperature and entropy the holostar will gradually lose particles by emission from its surface. The (exterior) time scale of this process will be comparable to the Hawking evaporation time scale $\propto r_h^3$ (see for example [7]). The (exterior) time for a photon to travel radially through the holostar is proportional to r_h^2. Therefore even comparatively small holostars are expected to have an evaporation time several orders of magnitude longer than their interior relaxation time.

This allows us, with the possible exception of near Planck-size holostars, to consider any spherical thin[12] shell within the holostar's interior to be in thermal equilibrium with its surroundings. Each shell can exchange particles, energy and entropy with adjacent shells on a time scale much shorter than the life-time of the holostar. Under these assumptions the thermodynamic parameters within each shell can be calculated via the grand canonical ensemble.

We mentally partition the holostar into a collection of thin spherical shells. The temperature scales as $1/\sqrt{r}$ and thus varies very slowly with r. For the chemical potential(s) let us assume a slowly varying function with r as well.[13] This assumption will be justified later. Under these circumstances the thickness of each shell δr can be chosen such, that it is large enough to be considered macroscopic, and at the same time small enough, so that the temperature, pressure and chemical potential(s) are effectively constant within the shell.

An accurate thermodynamic description has to take into account a possible potential energy of position. For the holostar a significant simplification arises from the fact, that the effective potential $V_{eff}(r)$ for the radial motion of massless, i.e. ultra-relativistic, particles is nearly constant, as can be seen from the following discussion. The equations of motion for ultra-relativistic particles within the holostar's interior were given by [7]:

$$\beta_r^2(r) + V_{eff}(r) = 1 \tag{23}$$

with

$$V_{eff}(r) = \frac{r_i^3}{r^3} \tag{24}$$

and

$$\beta_\perp^2(r) = \frac{r_i^3}{r^3} \tag{25}$$

$\beta_r(r)$ is the radial velocity of a photon, expressed as fraction to the local velocity of light in the (purely) radial direction. $\beta_\perp(r)$ is the tangential velocity of the photon, expressed as a fraction to the local velocity of light in the (purely)

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[12]With "thin" in the present context we mean small compared to the radial coordinate value r at a particular position. Note however, that even if δr is small, the *proper* radial thickness of a "thin" shell with radial extension δr can be huge, because the radial metric coefficient g_{rr} scales with r/r_0.

[13]Two natural choices present themselves: One is to to assume a constant, possibly zero, chemical potential. The other is to assume a chemical potential proportional to the temperature.

tangential direction. r_i is the turning point of the motion. For pure radial motion $r_i = 0$.

We find that for pure radial motion the effective potential is constant with $V_{eff}(r) = 0$. In the case of angular motion ($r_i \neq 0$) the effective potential approaches zero with $1/r^3$, i.e. becomes nearly zero very rapidly, whenever r is greater than a few r_i. Therefore, to a very good approximation we can regard the ultra-relativistic particles to move freely within each shell. Their total energy will only depend on the relativistic energy-momentum relation, not on the radial position.

With these preliminaries the grand canonical potential δJ of a small spherical shell of thickness δr for a gas of relativistic fermions at radial position r will be given by:

$$\delta J(r) = -T(r)\frac{f}{(2\pi\hbar)^3}\delta V \int \int \int d^3p \ln\left(1 + e^{-\frac{p-\mu(r)}{T(r)}}\right)$$

$$= -T^4\delta V\frac{f}{2\pi^2\hbar^3} \int_{z_{min}}^{z_{max}} z^2 \ln\left(1 + e^{-z+\frac{\mu}{T}}\right)dz$$

$z = p/T(r)$ is a dimensionless integration variable. Assuming that we have a low and high energy cut-off, the integration over p ranges from $p_{min} \approx \hbar/(2\pi r)$ to $p_{max} \approx p_{Pl} = \sqrt{\hbar}$.[14] $\mu(r)$ is the chemical potential at radial coordinate position r. $T(r)$ is the local temperature at this position. p_{PL} is the Planck-momentum, which is equal to the Planck-energy in units $c = 1$.

Note that even when the radial coordinate extension δr of the shell is small, the *proper* radial extension $\delta l = (r/r_0)^{1/2}\delta r$ of the shell will become quite large because of the large value of the radial metric coefficient in the holostar's outer regions.

Knowing the results presented at the end of this section it is not difficult to show that $z_{min} \approx 1/N^{1/4}$ and $z_{max} \approx N^{1/4}$, where N is the number of particles in the shell. With the exception of the central region of the holostar it is possible to choose the radial extension of the shell such that the number of particles within the shell, N, is macroscopic and at the same time $T(r)$ and $\mu(r)$ are constant to a very good approximation within the shell. For any holostar of macroscopic dimensions the number of particles in its non-central shells will be huge. Therefore the integration boundaries can be replaced to an excellent approximation by zero and infinity.

$$\delta J(r) = -T^4\delta V\frac{f}{2\pi^2\hbar^3} \int_0^\infty z^2 \ln\left(1 + e^{-z+\frac{\mu}{T}}\right)dz \qquad (26)$$

The proper volume of the shell δV is given by the volume element of equation (20).

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[14] We will see later, that these integration ranges can be replaced by 0 and ∞ to an excellent approximation, so the question, whether there truly is a low and/or high energy cutoff, as suggested by loop quantum gravity, or whether there is no such cut-off, as advocated by string-theory, is not relevant.

The ratio of chemical potential μ to local temperature T is assumed to be a very slowly varying function of r. In fact, we will see later that this ratio is virtually independent of r.[15] The ratio μ/T will be denoted by u, keeping in mind that u might depend on r:

$$u = \frac{\mu(r)}{T(r)}$$

The integral in equation (26) can be transformed to the following integral by a partial integration:

$$\delta J(r) = -T^4 \delta V \frac{f}{2\pi^2 \hbar^3} \frac{1}{3} \int_0^\infty z^3 n_F(z, u) dz \qquad (27)$$

where n_F is the mean occupancy number of the fermions:

$$n_F(z, u) = \frac{1}{e^{z-u} + 1} = \frac{1}{e^{\frac{p-\mu}{T}} + 1} \qquad (28)$$

Knowing the grand canonical potential δJ the entropy within the shell can be calculated:

$$\delta S(r) = -\frac{\partial(\delta J)}{\partial T} = \frac{f}{2\pi^2 \hbar^3} T^3 \delta V \left(\frac{4}{3} Z_{F,3}(u) - u Z_{F,2}(u) \right) \qquad (29)$$

By $Z_{F,n}$ the following integrals are denoted:

$$Z_{F,n}(u) = \int_0^\infty z^n n_F(z, u) dz \qquad (30)$$

Such integrals, which commonly occur in the evaluation of Feynman-integrals in QFT, can be evaluated by the poly-logarithmic function $Li_n(z)$:

$$Z_{F,n}(u) = -\Gamma(n+1) Li_{n+1}(-e^u) \qquad (31)$$

with $n+1 > 0$ and

$$Li_n(z) = \sum_{k=1}^\infty \frac{z^k}{k^n} \qquad (32)$$

For the derivation of the entropy the following identity has been used, which is easy to derive from the power-expansion of $Li_n(z)$.

$$\frac{\partial Z_{F,3}(u)}{\partial x} = 3 Z_{F,2}(u) \frac{\partial u}{\partial x} \qquad (33)$$

The pressure in the shell is given by:

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[15]This might not be exactly true at the radial coordinate position, where a phase transition takes place. I.e. where a particle species undergoes a transition from relativistic to non-relativistic motion.

$$P(r) = -\frac{\partial(\delta J)}{\partial(\delta V)} = \frac{f}{2\pi^2\hbar^3}T^4\frac{Z_{F,3}(u)}{3} \tag{34}$$

The total energy in the shell can be calculated from the grand canonical potential via:

$$\delta E(r) = \delta J - \left(T\frac{\partial}{\partial T} + \mu\frac{\partial}{\partial\mu}\right)\delta J = \frac{f}{2\pi^2\hbar^3}T^4\delta V Z_{F,3}(u) \tag{35}$$

The total number of particles within the shell is given by:

$$\delta N(r) = -\frac{\partial(\delta J)}{\partial\mu} = \frac{f}{2\pi^2\hbar^3}T^3\delta V Z_{F,2}(u) \tag{36}$$

The total energy per fermion within the shell is proportional to T, as can be seen by combining equations (35, 36):

$$\epsilon = \frac{\delta E}{\delta N} = \frac{Z_{F,3}(u)}{Z_{F,2}(u)}\,T(r) \tag{37}$$

ϵ only depends indirectly on r via u. We will see later that u is essentially independent of r, so that the mean energy per particle is proportional to the temperature with nearly the same constant of proportionality at any radial position r.

The entropy per particle within the shell can be read off from equations (29, 36):

$$\sigma = \frac{\delta S}{\delta N} = \frac{4}{3}\frac{Z_{F,3}(u)}{Z_{F,2}(u)} - u \tag{38}$$

Again, σ only depends on r via u.

The calculations so far have been carried through for fermions. It is likely, that the holostar will also contain bosons in thermal equilibrium with the fermions. The equations for an ultra-relativistic boson gas are quite similar to the above equations for a fermion gas. We have to replace:

$$n_F(z, u) \to n_B(z, u) = \frac{1}{e^{z-u} - 1} \tag{39}$$

$$Z_{F,n} \to Z_{B,n} = \int_0^\infty z^n n_B(z, u)dz \tag{40}$$

with $n + 1 > 0$ and

$$Z_{B,n} = \Gamma(n+1)\,Li_{n+1}(e^u) \tag{41}$$

Let us assume that the fermion and boson gases are only weakly interacting. In such a case the extrinsic quantities, such as energy and entropy, can be simply summed up. The same applies for the partial pressures.

The number of degrees of freedom of fermions and bosons can differ. The fermionic degrees of freedom will be denoted by f_F, the bosonic degrees of

freedom by f_B. In general, the different particle species will have different values for the chemical potentials. There are some restraints. Bosons cannot have a positive chemical potential, as $Z_{B,n}(u)$ is a complex number for positive u. Photons and gravitons, in fact all massless gauge-bosons, have a chemical potential of zero, as they can be created and destroyed without being restrained by a particle-number conservation law.

We are however talking of a gas of ultra-relativistic particles. In this case particle-antiparticle pair production will take place abundantly, so that we also have to consider the antiparticles. The chemical potentials of particle and anti-particle add up to zero: $\mu + \overline{\mu} = 0$. As bosons cannot have a positive chemical potential, the chemical potential of any ultra-relativistic bosonic species must be zero, i.e. $\mu_B = \overline{\mu_B} = 0$, whenever the energy is high enough to create boson/anti-boson pairs. This restriction does not apply to the fermions, which can have a non-zero chemical potential at ultra-relativistic energies, as both signs of the chemical potential are allowed. So for ultra-relativistic fermions we can fulfill the relation $\mu_F + \overline{\mu_F} = 0$ with non-zero μ_F.

For the following calculations it is convenient to use the ratio of the chemical potential to the temperature $u = \mu/T$ as the relevant parameter, instead of the chemical potential itself . If the number of degrees of freedom of fermions and bosons respectively, i.e. f_F and f_B is known, there are only two undetermined parameters in the model, u_F and β.[16]

In order to determine u_F and β one needs two independent relations. These can be obtained by comparing the holostar temperature and entropy to the Hawking temperature and entropies respectively.

Alternatively u_F can be determined without reference to the Hawking temperature law, solely by a thermodynamic argument. It is also possible to determine β by a theoretical argument as proposed in [5].

The thermodynamic energy of a shell consisting of an ultra-relativistic ideal fermion and boson gas is given by:

$$\delta E_{th} = \frac{F_E}{2\pi^2 \hbar^3} \delta V T^4 \tag{42}$$

with

$$F_E(u_F) = f_F(Z_{F,3}(u_F) + Z_{F,3}(-u_F)) + 2 f_B Z_{B,3}(0) \tag{43}$$

with the identities of the polylog-function and with $Z_{B,3}(0) = \pi^4/15$ one can express F_E as a quadratic function of u_F^2/π^2 [9]:

$$F_E(u_F) = 2 f_F \frac{\pi^4}{15} \left(\frac{15}{8} \left(1 + \frac{\pi^2}{u_F^2} \right)^2 + \frac{f_B}{f_F} - 1 \right) \tag{44}$$

We take the convention here, that f_F and f_B denote the degrees of freedom of one particle species, including particle and antiparticle. With this convention

[16]More generally, if the chemical potentials of the different particle species are different, one needs n-1 additional functional relations between the n independent chemical potentials.

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a photon gas ($g = 2$) is described by $f_B = 1$ (There are two photon degrees of freedom and the photon is its own anti-particle). All other particle characteristics, such as helicities, are counted extra. The total number of the degrees of freedom in the gas, i.e. counting particles and anti-particles separately, will be given by

$$f = 2(f_F + f_B) \tag{45}$$

The total energy of the holostar solution is given by the proper integral over the mass density. The proper energy of the shell therefore is:

$$\delta E_{BH} = \rho \delta V = \frac{\delta V}{8\pi r^2} = \frac{1}{2}(\beta\hbar)^{-\frac{1}{4}} r^{\frac{1}{2}} \delta r \tag{46}$$

Setting the two energies equal gives the local temperature within the holostar:

$$T^4 = \frac{\pi\hbar^3}{4F_E r^2} \tag{47}$$

Thus we recover the $1/\sqrt{r}$-dependence of the local temperature, at least if F_E is constant.

F_E is a function of f_F, f_B and u_F. We will see later, that u_F only depends on the ratio of f_F and f_B. Therefore in any range of r-values where the number of degrees of freedom of the ultra-relativistic particles (or rather their ratio) doesn't change, the local temperature as determined by equation (47) will not deviate from an inverse square root law.

If the temperature of equation (47) is inserted into equation (34), the thermodynamic pressure is derived as follows:

$$P(r) = \frac{1}{24\pi r^2} = \frac{\rho}{3}$$

This is the equation of state for an ultra-relativistic gas, as expected. Note, that the pressure doesn't exactly agree with the pressure of the holostar solution, although it is encouraging that the thermodynamic pressure at least has the right magnitude and r-dependence. In fact, the magnitude of the thermodynamic pressure is quite what is expected, when one takes into account that the thermodynamic derivation above is ignorant of the pressure anisotropy, treating all volume changes on an equal footing, independent of the direction of the change: Within the holostar the two tangential pressure components are zero, whereas the radial pressure is equal to $-1/(8\pi r^2)$. Therefore the "averaged" pressure over all three spatial dimensions is $-1/(24\pi r^2)$.

However, the thermodynamic pressure and the holostar pressure have opposite signs. This discrepancy cannot be resolved in the simple model discussed in this paper. This is not totally unexpected: The holostar is a *string* solution. Yet the thermodynamic model discussed in this paper assumes that the interior matter consists *exclusively* out of *radiation*. While it is likely that the interior matter will contain a significant radiation contribution at high temperatures,

it is unrealistic to assume that the dominant matter-type in the holographic solution - strings - is completely absent.[17]

Note also, that for the derivation of the main results of this paper, the particular form of the pressure is not essential, as was pointed out in section 2. Hopefully a satisfactory explanation for the discrepancy can be found in the future. A full explanation most likely will require a better understanding of the "string-nature" of matter not only at high, but also at low energies.

Another approach to resolve the problem is to replace the holostar-solution with a somewhat more general, yet similar solution to the field equations. For example, the equation of state could be modified (slightly), such as $P_r = (-1 + \delta(r))\rho$. Even with $\delta(r) = const$ one gets a significant tangential pressure component, as can be seen in [6]. Other modifications are thinkable. Yet one should be reluctant to modify the condition $g_{rr} = r/r_0$, as the results presented in this paper rely crucially on this condition.

4.1   Comparing the holostar's thermodynamic temperature and entropy to the Hawking result

By inserting the temperature derived in equation (47) into equation (29) we get the following expression for the thermodynamic entropy within the shell:

$$\delta S(r) = \left(\frac{F_E}{4\pi\beta}\right)^{\frac{1}{4}} \frac{F_S}{F_E} \frac{r\delta r}{\hbar} \tag{48}$$

with

$$F_S(u_F) = f_F \left(\frac{4}{3}\{Z_{F,3}(u_F)\} - u_F[Z_{F,2}(u_F)]\right) + 2f_B \frac{4}{3}(Z_{B,3}(0)) \tag{49}$$

We have used commutator [] and anti-commutator {} notation in order to render the above relation somewhat more compact.

Using the identities for the polylog function it is possible to express the above relation as a quadratic function of the variable u_F^2/π^2.

$$F_S = \frac{4}{3}F_E(u_F) - f_F \frac{\pi^4}{3} \frac{u_F^2}{\pi^2}\left(1 + \frac{u_F^2}{\pi^2}\right) \tag{50}$$

with F_E is given by equation (44)

By comparing the temperature (47) and the entropy (48) of the holostar solution derived in the context of our simple model to the Hawking entropy and

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[17]The inverse reasoning, that *all* matter in the holostar solution must be strings due to the interior string equation of state is incorrect. It is easy to construct the stress-energy tensor of a string gas by superposition of the stress-energy tensor for "normal" radiation and an equal vacuum contribution. Furthermore one can construct the interior stress-energy tensor of the holostar solution by superimposing the stress-energy tensor for a spherically symmetric charge-distribution with a vacuum contribution. See [9] or [5] for more details.

temperature, two important relations involving the two unknown parameters of the model u_F and β can be obtained.

We have already seen in section 3 that the holostar's temperature at infinity is proportional to the Hawking temperature. As can be seen from equation (47) this general result remains unchanged in the more sophisticated thermodynamic analysis, as long as the quantity $F_E(u_F, f_F, f_B)$ can be considered to be nearly constant. We will see later, that the value of u_F only depends on the ratio f_B/f_F, so that $F_E = const$ whenever the number of fermionic and bosonic degrees of freedom don't change. In order to determine F_E we can set the temperature at the holostar's surface equal to the blue shifted Hawking temperature at the holostar's surface, which can be obtained by multiplying the Hawking temperature (at infinity) with the red-shift factor z of the surface given in Eq. (15). We find:

$$T^4 = T_{BH}{}^4 z^4 = \frac{\hbar^4}{2^8 \pi^4 r_h{}^4} \cdot \frac{r_h^2}{\beta \hbar} = \frac{\hbar^3}{2^8 \pi^4 \beta r_h{}^2} \tag{51}$$

Comparing this to equation (47) we find:

$$\frac{F_E}{4\pi\beta} = (2\pi)^4 \tag{52}$$

This is an important result. It relates the fundamental area $4\pi r_0^2 = 4\pi\beta\hbar$ to the thermodynamic parameters of the system, i.e. the number of degrees of freedom and the chemical potential of the fermions.

Another important relation is the ratio F_S/F_E in the interior holostar spacetime, which can be obtained by comparing the Hawking entropy of a black hole with thermodynamic entropy of the holostar's interior constituent matter.

The entropy of the holostar can be calculated by integrating equation (48). We will assume that $F_E/\beta = const$, as follows from equation (52), and that $F_S/F_E = const$, which will be justified shortly. If this is the case, the integral can be performed easily:

$$S = \int_0^{r_h} \delta S(r) dV = \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} \frac{1}{2\pi} \frac{F_S}{F_E} \frac{A}{4\hbar} \tag{53}$$

with

$$A = 4\pi r_h{}^2$$

Setting this equal to the Hawking entropy, $S_{BH} = A/(4\hbar)$, and using equation (52) we find the important result:

$$\frac{F_S}{F_E} = 1 \tag{54}$$

Writing out the above equation we get:

$$\frac{f_F \left(\frac{4}{3} \{Z_{F,3}(u_F)\} - u_F[Z_{F,2}(u_F)] \right) + 2f_B \left(\frac{4}{3} Z_{B,3}(0) \right)}{f_F \{Z_{F,3}(u_F)\} + 2f_B Z_{B,3}(0)} = 1 \tag{55}$$

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which can be simplified to

$$\frac{u_F[Z_{F,2}(u_F)]}{\{Z_{F,3}(u_F)\} + 2\frac{f_B}{f_F}Z_{B,3}(0)} = \frac{1}{3} \tag{56}$$

Using the identities for the polylog function one can reduce the above equation to a very simple quadratic equation in the variable u_F^2/π^2:

$$\left(1 + \frac{u_F^2}{\pi^2}\right)\left(1 - 3\frac{u_F^2}{\pi^2}\right) + \frac{8}{15}\left(\frac{f_B}{f_F} - 1\right) = 0 \tag{57}$$

The important message, which can be seen already from equation (56) is, that whenever the bosonic and fermionic degrees of freedom - or rather their ratio f_B/f_F - is known, u_F can be calculated. Knowing u_F, β can be determined via (52). Thus the two relations (52, 54) allow us to determine all free parameters of the model, whenever the number of particle degrees of freedom, f_F and f_B are known.

4.2 An alternative derivation of the relation $F_S/F_E = 1$

Before discussing the specifics of the thermodynamic model, I would like to point out another derivation of equation (54), which does not depend on the Hawking result. This alternative derivation only depends on the following fundamental thermodynamic relation

$$\frac{\delta S}{\delta E}T = 1 \tag{58}$$

and on the fact, that the holostar's interior matter state is completely rigid, i.e. the interior matter state at any particular radial position depends only on r, but not on the overall size of the holostar.

Consider a process, where an infinitesimally small spherical shell of matter is added to the outer surface of the holostar. This process doesn't affect the inner matter of the holostar, as the interior matter-state of the holostar at a given radial coordinated position r does not depend in any way on the size of the holostar or on any other global quantity. Therefore, when adding a new layer of matter we don't have to consider any interaction, such as heat-, energy- or entropy-transfer between the newly added matter layer and the interior matter.[18] It is an adiabatic process, for which we can calculate the entropy-change of the whole system via equation (58). Let r be the radial position of the holostar's surface. The entropy of the newly added shell is given by equation (48), its energy by equation (46), and its temperature by equation

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[18] This statement implicitly assumes, that we can neglect the effect of the boundary membrane, which might be an oversimplification. When we place a new layer of matter with radial extension dr at the former boundary r of the holostar, the boundary membrane moves throughout the newly added layer to its new position at $r + dr$. It is not altogether clear, whether this process is adiabatic. However, in section 13 arguments are given, that the membrane has zero entropy, so that the assumption, that the different initial and final states of the membrane have no effect on the thermodynamics of the process, seems not too far fetched.

(47). One finds that the thermodynamic relation (58) is only fulfilled, when $F_S = F_E$. We have derived equation (54) only from thermodynamics.

4.3   A closed formula for u_F and some special cases

The chemical potential per temperature u_F can be determined by finding the root of equation (57). The value of u_F depends only on the ratio of fermionic to bosonic degrees of freedom.[19]. Let us denote the ratio of the degrees of freedom by

$$r_f = \frac{f_B}{f_F} \tag{59}$$

Then u_F is given by:

$$\frac{u_F^2}{\pi^2} = \frac{2}{3}\sqrt{1 + \frac{2}{5}(r_f - 1)} - \frac{1}{3} \tag{60}$$

For $r_f = 0$ (only fermions) we find the following result:

$$u_F = \pi \sqrt{\sqrt{\frac{4}{15}} - \frac{1}{3}} = 1.34416 \tag{61}$$

For $r_f = 1$ (equal number of fermions and bosons) we get:

$$u_F = \frac{\pi}{\sqrt{3}} = 1.8138 \tag{62}$$

From equation (60) one can see that u_F is a monotonically increasing function of r_f. It attains its minimum value, when there are no bosonic degrees of freedom, i.e. $f_B = r_f = 0$. When the bosonic degrees of freedom vastly exceed the fermionic degrees of freedom, u_F can - in principle - attain high values. For large r_f we have $u_F \propto (r_f - 1)^{1/4}$. For all practical purposes one can assume that the number of bosonic degrees of freedom is not very much higher than the number of fermionic degrees of freedom. This places u_F in the range $1.34 < u_F < 3$.

It is important to notice, that equation (56) only has a solution when the number of fermionic degrees of freedom, f_F, is non-zero, whereas f_B can take arbitrary values for any non-zero f_F. Therefore at least one fermionic (massless) particle species with a non-vanishing chemical potential proportional to the local radiation temperature is necessary, if the interior mass-energy-density $1/(8\pi r^2)$ of the holostar is to be in thermodynamic equilibrium.

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[19] and on the constant ratio F_S/F_E, which has been shown to be unity for the interior holostar solution

4.4 Thermodynamic relations, which are independent from the Hawking formula

If u_F is known, all thermodynamic quantities of the model, such as $F_E(u_F)$ and $F_N(u_F)$ etc. can be evaluated. Note that in order to determine u_F we only needed the relation $F_E = F_S$, whose derivation didn't require the Hawking temperature/entropy relation. Yet in order to fix β via equation (52) we had to compare the holostar's temperature (or entropy) to the Hawking-result. Therefore the particular relation between β and F_E derived in equation (52) is tied to the the validity of the Hawking temperature formula.

Although there is no doubt that the Hawking temperature of a large black hole must be inverse proportional to its mass[20], the exact numerical factor has not yet been determined experimentally and thus might be questioned. For example, the Hawking entropy/temperature could be subject to a moderate rescaling[21], so it is worthwhile to know what thermodynamic relations in the interior holostar space-time are independent from the Hawking formula. The following derivations only make use of equation (54), i.e. $F_E = F_S$.

Knowing u_F from equation (60) the entropy per particle, σ, can be easily calculated by equations (29, 36):

$$\sigma = \frac{\delta S}{\delta N} = \frac{F_S}{F_N} = \frac{F_E}{F_N} \tag{63}$$

with

$$F_N(u_F) = f_F(Z_{F,2}(u_F) + Z_{F,2}(-u_F)) + 2f_B Z_{B,2}(0) \tag{64}$$

The energy per relativistic particle is given by equations (42, 70). We find, just as in the previous section, that the mean particle energy per temperature is constant[22] and equal to the mean entropy per particle:

$$\epsilon = \frac{\delta E}{\delta N} = \frac{F_E}{F_N} T = \sigma T \tag{65}$$

σ only depends on the number of degrees of freedom of the ultra-relativistic bosons and fermions in the model. In fact, σ only depends on the ratio $r_f = f_B/f_F$ and is a very slowly varying function of this ratio. Figure 1 shows the dependence of σ on r_f. In section 8 the values of the entropy per particle σ, the ratio of chemical potential to temperature of the fermions u_F, and other

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[20]This already follows from the Bekenstein-argument, that the entropy of a black hole should be proportional to the surface of its event horizon.

[21]There are two possible effects which could influence the value $4\hbar$ in the denominator of the Hawking entropy-area formula $S = A/(4\hbar)$. First $4\hbar$ is a "fundamental area". Its value depends on Newton's constant. It has been speculated, that Newton's constant might undergo a (finite) renormalization depending on the energy scale. Second, the holostar's membrane isn't situated at the gravitational radius of the holostar, but roughly a Planck coordinate distance outside. Therefore the holostar's temperature at infinity might be slightly lower (and its entropy higher) than the Hawking result, which assumes an exterior vacuum space-time right up to the horizon. The second effect should be quite negligible for large holostars.

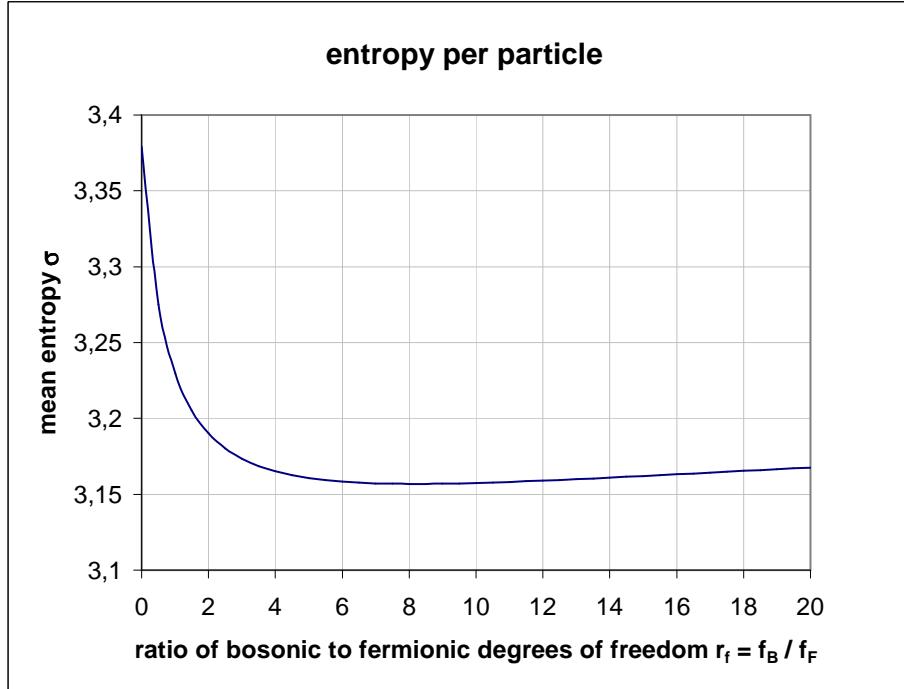[22]at least as long as f_F and f_B remain constant

Figure 1: mean entropy per particle of the ultra-relativistic fermions in the holographic solution as a function of the ratio of bosonic to fermionic degrees of freedom $r_f = f_B/f_F$

interesting thermodynamic parameters are tabulated for several values of f_F and f_B.

The relation $\epsilon = \sigma T$, which relates the mean energy per particle to the mean entropy times the local radiation temperature can be viewed as the *fundamental thermodynamic characteristic* of the holostar. Keep in mind that this relation is only valid for the *mean* energy per particle and the *mean* entropy per particle, evaluated with respect to all particles. It isn't fulfilled for the bosonic and fermionic species individually. In general, except for the special case $f_B = 0$, we have $\epsilon_B \neq \sigma_B T$ and $\epsilon_F \neq \sigma_F T$.

The relation $\epsilon = \sigma T$, which is equivalent to $F_S = F_E$, has the remarkable side-effect, that the free energy is identical zero in the holostar solution:

$$F = E - ST = N(\epsilon - \sigma T) = 0 \qquad (66)$$

Usually a closed system has the tendency to minimize it's free energy, which is a compromise between minimizing it's energy and maximizing it's entropy. The holostar is the prototype of a closed system. It is a self-gravitating static solution to the Einstein field equations. It's only form of energy-exchange with

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the outer world is through Hawking-radiation, which is an utterly negligible mode of energy-exchange for a large holostar. In this respect it is remarkable that the holostar solution minimizes the free energy to zero, e.g. the smallest possible value that a sensible measure of energy in general relativity can have[23]. One can speculate on the basis of this result, whether the free energy in general relativity might be more than a mere book-keeping device.

With the help of equation (54), but not using equation (52), the entropy within the shell can be expressed as:

$$\delta S(r) = \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} \frac{r\delta r}{\hbar} \tag{67}$$

If the total entropy of the holostar, i.e. the integral over the entropy-contributions of the respective shells, is to be proportional to the Hawking entropy of a black hole with the same gravitational radius, F_E/β must be constant. Integration of equation (67) gives the result:

$$S = \frac{1}{2\pi} \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} \frac{A}{4\hbar} \tag{68}$$

The Hawking result is reproduced, whenever:

$$\omega = \frac{1}{2\pi} \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} = 1 \tag{69}$$

ω, which depends on the ratio F_E/β, is the constant of proportionality between the holostar entropy and the Hawking entropy. Setting $\omega = 1$ is equivalent to equation (52), which fixes β with respect to the Hawking temperature. If the Hawking entropy/temperature formula have to be rescaled, ω is nothing else than the (nearly constant) scale factor. Therefore let us express all thermodynamic relations in terms of ω.

The number of particles within the shell is given by equation (36), which is extended to encompass the bosonic degrees of freedom:

$$\delta N(r) = \frac{F_N}{F_E} \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} \frac{r\delta r}{\hbar} = \frac{\omega}{\sigma} \frac{2\pi r\delta r}{\hbar} \tag{70}$$

The total number of particles is given by a simple integration, assuming that $\omega = const$:

$$N = \left(\frac{F_E}{4\pi\beta} \right)^{\frac{1}{4}} \frac{1}{2\pi} \frac{1}{\sigma} \frac{A}{4\hbar} = \frac{\omega}{\sigma} \frac{A}{4\hbar} \tag{71}$$

Therefore, as derived in the previous section, the total number of particles within the holostar is proportional to its surface area, whenever $F_E/\beta = const$ and $\sigma = const$.

The temperature of the holostar at infinity is given by

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[23]In general relativity the total energy is always positive

$$T_\infty = T(r_h)\sqrt{g_{tt}(r_h)} = 2\pi \left(\frac{4\pi\beta}{F_E}\right)^{\frac{1}{4}} \frac{\hbar}{4\pi r_h} = \frac{1}{\omega}\frac{\hbar}{4\pi r_h} \tag{72}$$

Again, if we set $\omega = 1$ we get the Hawking temperature. The important result is, that ω could in principle take on any arbitrary (nearly constant) value. This is possible, because the factor in the temperature is just the inverse as the factor in the entropy. As is well known from black hole physics, any constant rescaling of the Hawking entropy must necessarily rescale the temperature such, that the product of temperature and entropy is equal for the scaled and unscaled quantities, i.e. ST must be unaffected by the rescaling. This is necessary, because otherwise the thermodynamic identity

$$\frac{\partial S}{\partial E}T = 1$$

would not be fulfilled in the exterior space-time. (In the exterior space-time the energy E is fixed and is taken to be the gravitating mass $M = r_h/2$ of the black hole.)

As can be seen from equations (68, 72), entropy and temperature at infinity of the holostar fulfill the rescaling condition. Furthermore, entropy and temperature at infinity are exactly proportional to the Hawking temperature and entropy. This result is not trivial. It depends on the holostar metric, which has just the right value at the position of the membrane, so that the temperature at infinity scales correctly with respect to the entropy.

4.5 Relating the local thermodynamic temperature to the Hawking temperature

Now we are ready to set $\omega = 1$, which gives us the desired relation between β and F_E, as already expressed in equation (52).

With $\omega = 1$, the local thermodynamic temperature of any interior shell can be expressed solely in terms of β. It turns out to be equal to the expression in equation (22) of the previous section:

$$T^4 = \frac{\hbar^3}{(4\pi)^4\beta}\frac{1}{r^2} \tag{73}$$

or

$$T^4 A = \frac{1}{\beta}\left(\frac{\hbar}{(4\pi)}\right)^3 = const$$

5 A measurement of the Hawking temperature

In the previous section the internal temperature of the holostar has been derived by "fixing" it with respect to the Hawking temperature. Although Hawking's calculations are robust and there appears to be no reason, why the Hawking

equation should be modified - at least for large black holes[24] - it has been speculated whether the factor in the entropy-area law (or in the temperature formula) might take a different value. A single measurement of the Hawking temperature (or entropy) of a large black hole could settle the question. However, with no black hole available in our immediate vicinity and taking into account the extremely low temperatures of even comparatively small black holes, there appeared to be no feasible means to measure the Hawking entropy or temperature of a black hole directly or indirectly.

It would be of high theoretical value, if the Hawking temperature/entropy formula could be verified (or falsified) by an explicit measurement. The holostar provides such a means.

For this purpose let us assume, that the Hawking temperature formula were modified by a constant factor, i.e

$$T = \frac{1}{\omega} \frac{\hbar}{4\pi r} \tag{74}$$

where ω is a dimensionless factor, whose value can be determined experimentally.

If we set the temperature of the holostar equal to the modified Hawking temperature we get the following result for F_E:

$$\frac{F_E}{4\pi\beta} = (2\pi\omega)^4 \tag{75}$$

The local temperature within the holostar is then given by equation (47):

$$T^4 = \frac{\hbar^3}{2^8\pi^4\beta r^2\omega^4} = \frac{1}{\omega^4} \frac{\hbar^3}{2^5\pi^3\beta}\rho \tag{76}$$

$\rho = 1/(8\pi r^2)$ is the total (local) energy density of the matter within the holostar. The above equation can be solved for ω:

$$\omega^4 = \frac{\hbar^3}{2^5\beta\pi^3} \frac{\rho}{T^4} \tag{77}$$

The local radiation temperature T and the total local energy density ρ within a holostar are both accessible to measurement. Note that the local temperature within a holostar is much easier to measure than its (Hawking) temperature at infinity: The local interior temperature only scales with $1/\sqrt{M}$, whereas the temperature at infinity scales with $1/M$. Therefore even a very large holostar will have an appreciable interior local radiation temperature, although its Hawking temperature at infinity will be unmeasurable by all practical means.

In order to determine ω the value of β need to be known. In [5] the following formula for β has been suggested:

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[24]The only ingredient in Hawking's derivation is the propagation of a quantum field in the exterior vacuum space-time of a black hole. Both concepts (quantum field in vacuum; exterior space-time of a black hole) are very accurately understood.

$$\frac{\beta}{4} = \frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}} \tag{78}$$

α is the running value of the fine-structure constant, which depends on the local energy scale. Note that the above relation for β has not been derived rigorously in [5], but was suggested by analogy, i.e. by extrapolating the (exact) formula derived for an extremely charged holostar to the rotating case. Angular momentum was introduced in rather straightforward way, giving the correct formula for a Kerr-Newman black hole in the macroscopic limit and the correct formula for the non-rotating case ($J = 0$). The extrapolation formula then was applied to a *microscopic* object, a spin-1/2 extremely charged holostar of minimal mass, in order to obtain equation (78). One must keep in mind though, that in principle there are several different choices, which give the correct macroscopic limit, but which might differ in their microscopic predictions.

We wan't to apply equation (78) in order to derive the value of r_0 which determines the *interior* radial metric coefficient of a large holostar, $g_{rr} = r_0/r$. The implicit assumption which lies at the heart of equation (78), is that $r_0^2 = \beta\hbar$ is a *universal* quantity, not dependent on the nature of the system in question and only - moderately - dependent on the energy-scale. It requires quite a leap of faith to do this. However, this assumption will be - at least partly - theoretically justified by the discussion in section 9 and is backed by some experimental evidence, as will be shown in the same section.

If we assume $r_0^2 = \beta\hbar$ to be universal, it is not quite clear how the value of α should be interpreted when the local energy-densities and temperatures are high. In the original "derivation" of equation (78) α referred to the value of the fine-structure constant for a single charged spin-1/2 minimal mass holostar, which was identified with an electron *at rest*. The original "derivation" therefore refers to a low-energy situation, where the electromagnetic field is the only long-range field besides gravity.

Due to the appearance of α in the formula for r_0^2 it is suggestive to interpret r_0 as a running length scale, which depends on the energy E via $\alpha(E)$. This means that for high temperatures r_0 is expected to increase with energy as a function of $\alpha(E)$. This makes sense, because we have already seen, that r_0^2 is proportional to the effective degrees of freedom, which are also known to increase at high energies. Therefore, if we treat r_0^2 as a universal quantity, the only sensible way is to interpret α as the running value of the relevant coupling constants depending on the energy scale. Whenever the energy scale becomes comparable to the strong interaction scale or the electro-weak unification scale, the other coupling constants have to be properly accounted for.

With this interpretation, whenever α is small, such as for the typical energies encountered today, it can be set to zero in the above equation to a very good approximation, so that $\beta \approx 4\sqrt{3/4}$.

Let us now make the assumption, that we live in a large holostar. In [7] several observational facts have been accumulated which suggest that such a claim is not too far fetched. Then the local radiation temperature will be nothing

else than the microwave-background temperature and the total (local) energy density will be the total matter density of the universe at the present time (= present radial position). Both quantities have been determined quite precisely in the recent past. With the following value for the temperature of the microwave background radiation

$$T_{CMBR} = 2.725K$$

and with the total matter density determined from the recent WMAP-measurements [3]

$$\rho = 0.26\,\rho_c = 2.465 \cdot 10^{-27}\frac{kg}{m^3}$$

and with β determined from equation (78) using the present (low energy) value of the fine-structure constant, α,

$$\beta = 3.479$$

we find:

$$\omega^4 = 1.0116 \qquad\qquad (79)$$

or

$$\omega = 1.003$$

If we set the fine-structure constant to zero, i.e. $\beta = 4\sqrt{3/4}$, the agreement is almost as good: $\omega = 1.004$. The very high accuracy suggested in the above results is somewhat deceptive. With T known to roughly 0.1% the error in ω will be dominated by the uncertainty in ρ. A conservative estimate for this uncertainty should be roughly 5%. Taking the fourth square root suppresses the relative error by roughly a factor of four, so that the error in ω will be roughly 1%. Therefore, within the uncertainties of the determination of ρ and T the Hawking-entropy formula is reproduced to a remarkably high degree of accuracy of roughly 1%.

6 A scale invariant reformulation for the Hawking entropy

In this section I briefly discuss some implications of a (moderate) rescaling of the gravitational constant G at high energies, i.e. whether the formula for the entropy, particle number, local temperature and temperature at infinity derived in the last sections are invariant under a rescaling of G, or if they can be reformulated in an invariant manner.

Before I do this, let me clarify my own position on this issue. If one believes in the field equations of general relativity, which are independent from a particular energy scale, then one cannot possibly come to think that G should be an

energy-dependent quantity. Much of the beautiful geometric interpretation of the field equations were lost, if G were variable. In effect, a variable G would completely change the internal structure of the theory. The constancy of G is an essential requirement in the Einstein equations. Yet these equations have only been verified at low energies. There is a common understanding, that at high energies the field equations might have to be modified. The best suited candidate theory for such a modification is string-theory. Does string-theory require G to be running? I doesn't necessarily appear so. G, or rather a combination of G and the other constants $(c, \hbar)$ with the dimension of length is needed to define the dimensionless string coupling. Yet many string-theorists tend to view the string-coupling as a quantity that does not run. As long as there is no hard theoretical evidence for a running G, I prefer the purist position, that G is a true constant of nature, as the other two constants c and $\hbar$.

Yet there is no proof for this position, so it is instructive to reflect on the possible implications that a running G might have.

The notation that was used so far is somewhat inconvenient for the purpose of this section, because $G = 1$ was assumed in the previous sections. It is not difficult to re-introduce G. Any occurrence of $\hbar$ has to be replaced by $\hbar G$ and enough powers of $\hbar$ have to be inserted into the equations, so that the dimensions come out correct. The convention $c = 1$ will be retained, i.e. length and time are interchangeable. G then has dimensions $[m/kg]$, i.e. allows us to express any mass as a length and vice versa. $\hbar$ is complementary to G. Its dimensions are $[m\,kg]$, so that via $\hbar$ any length can be expressed as an inverse mass (or energy) and vice versa.[25]

In contrast to G, which might behave like a running coupling constant, c should be thought of as a true constants of nature, independent of the energies of the respective interactions.[26] $\hbar$ also should be viewed as a true constant of nature, independent of the energy scale.[27]

From G and $\hbar$ two important combinations can be formed. In units $c = 1$ the quantity $\hbar G$ has the dimensions of area, i.e. can be considered as a fundamental area. Furthermore the quantity $\hbar^2/(\hbar G) = \hbar/G$ has dimensions of energy (or mass) squared, so that $\hbar G$ can alternatively be regarded as an inverse energy squared.[28]

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[25]There appears to be some sort of duality between $\hbar$ and G: In a certain sense $\hbar$ and G are complimentary with respect to transforming a length- into an energy scale. Whereas $\hbar$ transforms length into inverse energy, G transforms length into energy directly.

[26]The constancy of c is linked to one of the most important space-time symmetries, i.e. Lorentz-invariance, which is one of the basic building blocks of classical and quantum physics. Nobody should give up Lorentz-invariance lightly.

[27]$\hbar$ arises from rotational symmetry which dictates the quantization of angular momentum in half-integer steps of $\hbar$. If $\hbar$ were variable, we would have a severe problem with the conservation law for angular momentum.

[28]Note that the cross-sectional area σ of the high energy interactions between particles in the standard model follows an inverse square law in the energy, i.e. $\sigma \propto \hbar^2/E^2$ when the rest-masses of the particles are negligible. The cross-sectional area of a black hole (or holostar), however, scales with E^2, so there is some sort of duality as well. This has some resemblance to string theory, where the winding modes and the vibrational modes also have a different (proportional vs. inverse proportional) dependencies on the radius of the circular dimension.

The equations derived in the previous section depend on the value of the gravitational constant G, which enters into the definition of the fundamental area $\hbar \rightarrow \hbar G/c^3$. The formula are not necessarily invariant under a rescaling of G. For example, the entropy is given by $S = A/(4\hbar G)$. Whenever the value of G is modified, the entropy of the system changes due to the change in the fundamental area (unless the measurement process of the area A somehow compensates the change in G). As long as G is constant there is no problem. But if G behaves similar to the coupling constants in gauge-theories, it might be subject to a (moderate) rescaling at high energies.[29]

Assuming that G might vary can regarded as a new type of symmetry. Why should G be considered as variable? Obviously G has a definite value, at least at the typical energy scales encountered today. On the other hand, in the purely classical sector of general relativity, i.e. if we disregard quantum effects (such as the discreteness of the geometry and the existence of a fundamental area and fundamental particles), the gravitational constant G is just a convention which depends on the chosen length or time scale. If macroscopic classical general relativity is to be truly relational in the full Machian sense, i.e. only the relative positions and sizes of "real macroscopic objects" count and no fundamental reference scale (such as the minimum boundary area of a fundamental quantum of geometry or particle) is given, the macroscopic phenomena predicted by general relativity and statistical thermodynamics of great numbers should be scale invariant, i.e. exactly the same, whatever length scale we chose and whatever the particular value of the gravitational constant G with respect to this length scale may be.

Therefore let us try to devise a reformulation of the results of the previous sections so that all of the relevant phenomena, i.e. the relations between entropy, local temperature, (Hawking) temperature at infinity, metric etc. can be described in a way that is essentially independent of G. Or formulated somewhat differently: We are looking for an additional symmetry in the equations which allows G (or $\hbar G$) to vary without changing the physical results predicted by the equations.

How can such a scale invariant reformulation be found? For this purpose lets take a critical look on the Hawking entropy formula. The Hawking entropy is equal to the surface area of a black hole, measured (for example) in square meters, divided by four times the Planck area, $4\hbar G$. Via G the Planck area can be expressed in square meters as well, so that a dimensionless quantity arises for the entropy. The problem is, that the area A measured in square meters,

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In string theory, a state with a particular winding and vibration number is indistinguishable from a state, where winding and vibration number are interchanged.

[29]Note that Einstein's theory of general relativity is difficult to quantize, if the "normal" quantization procedure for a gauge-invariant theory on a fixed space-time background are used. Only by exploiting the (additional) symmetry of diffeomorphism invariance could a non-perturbative quantization of gravity be achieved. Quite unexpectedly the (non-perturbative) quantization turned out to be finite to all orders. It might well be that the diffeomorphism-invariance of Einstein's theory of gravitation provides a greater "protection" for the gravitational (coupling) constant G against renormalization than the gauge symmetries in the typical gauge theories over a fixed background, so that in fact G might be constant at all energy scales.

and the Planck area $\hbar G$ expressed in square meters, might transform differently under a change of G: A shift in G will change the fundamental Planck-area, but according to our common understanding of quantum theory such a shift shouldn't affect our definition of the meter: The meter is defined via the second which is linked to an atomic transition with a very sharply defined frequency. The frequency of the atomic transition is a pure quantum effect and therefore should not be affected by a change in G.[30] Therefore the measured surface area of a black hole or holostar (in square meters) should be independent of G. On the other hand, the Planck area $\hbar G$ depends explicitly on G. If G can vary, one would assume that the divisor for the area in the entropy formula should rather be a "fundamental area", whose measured value (for example in square meters) is independent on a variation of G.

Is there an alternative to the divisor $4\hbar G$ in the Hawking formula? For the holostar solution we have the so called fundamental length parameter, r_0, from which a fundamental area r_0^2 can be formed. r_0 is defined as the maximum of the radial metric coefficient g_{rr} which attains its maximum at the surface of the holostar, i.e. $g_{rr\,max} = g_{rr}(r_h) = r_h/r_0$. The area r_0^2 is related to the Planck area via $r_0^2 = \beta \hbar G$. This is a significant improvement, because of the factor β. We can compensate any change of the gravitational constant G by a variation of β. Furthermore, the value of β can be determined without reference to G, as can be seen from equation (84). A change in G therefore doesn't affect the measurement of β, which seems just what is needed. Also note, that β appears to be just in the right range, i.e. $\beta = r_0^2/(\hbar G) \approx 4$, so that the deviation to the Hawking formula wouldn't be large. In the light of the above discussion we are lead to modify the Hawking entropy formula as follows:

$$ S = \frac{4\pi r^2}{r_0^2} = \frac{1}{G}\frac{4\pi r^2}{(\beta\hbar)} = \left(\frac{G}{\beta\hbar}\right)16\pi M^2 \tag{80} $$

$\beta = 4$ corresponds to the usual Hawking entropy-area law.

With this definition F_E can be expressed in terms of β, by setting the entropy of equation (68) equal to (80).

$$ \frac{F_E}{4\pi\beta} = \left(\frac{8\pi}{\beta}\right)^4 \tag{81} $$

The local temperature is given by equation (47)

$$ T^4 = \frac{1}{G}\frac{(\beta\hbar)^3}{2^{16}\pi^4 r^2} = \left(\frac{\beta\hbar}{G}\right)^3\frac{1}{2^{18}\pi^4 M^2} \tag{82} $$

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[30]This statement is only correct for quantum theory on a fixed background. There are several objections to the above statement, both practical and in principle: First, quantum field theory on a fixed background is not diffeomorphism invariant, and therefore should be regarded rather as an - albeit excellent - approximation to the real physical phenomena. Second, the transition frequency of an atomic clock depends on the value of the fine-structure constant, which runs. However, this dependence may be calculated in the context of QFT. Lastly there is the practical problem to construct an atomic (or any other accurate) clock at high energies.

M is the gravitating mass of the holostar enclosed by the radius r. The Hawking temperature reads:

$$T_\infty = \frac{(\beta\hbar)}{16\pi r_h} = \left(\frac{\beta\hbar}{G}\right)\frac{1}{32\pi M} \tag{83}$$

All the other equations of the previous section remain the same. A very interesting thing has happened. With our particular ansatz for the entropy (see equation (80)), β always appears in combination with $1/G$ (or rather $\hbar/G$) in all equations, when the quantities are expressed in terms of the gravitating mass M (i.e. in "mass-coordinates"). On the other hand, when the quantities are expressed in terms of the radial coordinate r, i.e. in terms of length, β always appears in combination with $\hbar$.

Note also that equation (82) relates the local temperature to the local energy density $\rho = 1/(8\pi r^2 G)$. If expression (82) is expressed in terms of the measurable quantities ρ and T, the relation turns out to be independent of G:

$$T^4 = \frac{1}{G}\frac{(\beta\hbar)^3}{2^{16}\pi^4 r^2} = \frac{(\beta\hbar)^3}{2^{13}\pi^3}\rho \tag{84}$$

In fact, all thermodynamic equations, such as $\partial S/\partial E = 1/T$ are independent of G.

We can now analyze what happens if G (or $\hbar$) vary. Let us first assume $G = const$, but $\hbar$ variable. Any variation in $\hbar$ can then be compensated by a respective change in β, as long as $\hbar\beta$ remains constant. I.e β must change inverse proportional to $\hbar$. On the other hand, if G varies and $\hbar$ is constant, a change in G will not affect the equations (expressed in "mass-coordinates"), whenever β varies proportional to G, as G always appears in combination with $\beta\hbar/G$.

Again there is some sort of duality between the quantities G and $\hbar$.

We are confronted with two different choices for the Hawking-entropy formula. Either we stick to the classical result $S = \frac{A}{4\hbar}$ or we chose equation (80). The equations for the internal temperature within the holostar, the (Hawking) temperature at infinity and the number of degrees of freedom F_E will be affected by the choice, as can be seen from equations (83, 82, 81).

What particular choice we should make, depends on the nature of the gravitational constant G. If G is a universal constant of nature, independent of the energy scale, i.e. not subject to a (moderate) renormalization at high energies, we should use the original Hawking entropy formula. However, if G undergoes renormalization at high energies, it appears more appropriate to pick the more general approach with β in the divisor of the entropy formula.

From an aesthetic point of view, the second choice has some merit. However, not aesthetics, but measurements will have to decide the issue. In the previous section it has been demonstrated, that under the assumption that G is constant, the Hawking-entropy formula is correct to better than 1%. If the entropy formula is to be modified according to the discussion of this section, the value of β can be determined experimentally along the same lines as in the previous

section. Equation (84) gives a relation between the local radiation temperature and the total local energy density within the holostar:

$$T^4 = \left(\frac{\beta\hbar}{8\pi}\right)^3 \frac{\rho}{16} \tag{85}$$

This can be solved for β:

$$\beta = \left(\frac{(2T)^4}{\rho}\right)^{\frac{1}{3}} \frac{8\pi}{\hbar} \tag{86}$$

With $T_{CMBR} = 2.725K$ and $\rho_{WMAP} = 2.465 \cdot 10^{-27} \frac{kg}{m^3}$ we find:

$$\beta = 4.17 \tag{87}$$

When this value is compared with equation (78), it turns out far too high for the low temperature region of the universe, where the fine-structure constant α is small. Furthermore, it doesn't fit well into the framework developed in [5, 7]. Therefore at the current state of knowledge it appears more likely, that the Hawking entropy formula is correct for all scales, which in turn can be interpreted as indication, that the gravitational constant G is a true constant of nature such as $\hbar$ and c, i.e. independent of the energy scale.

On the other hand, if the total matter density ρ would turn out to be roughly 13% higher than the WMAP result, we would get $\beta = 4$, which would make both approaches essentially indistinguishable.

7 Are the gravitational constant, the fine-structure constant and the electron mass related?

In this section I propose a relation between the gravitational constant G, the fine-structure constant α and the electron-mass m_e. This proposal most likely will be considered by any conservative researcher as a blind leap into the dark, rather fuelled by the faith of the foolish than by the wisdom of knowledge. And not much can be said against such a point of view: The relationship proposed in equation (96) is based more on "playing with numbers" than on truly convincing physical arguments. Therefore it is quite probable that (96) will suffer the fate of the vast majority of proposals of a similar kind, such as the numerous formula given for the value of the fine-structure constant, which even appear in regular intervals today.[31] On the other hand, the gravitational constant is the only fundamental constant of nature that hasn't been measured to a high degree of precision. Therefore from the viewpoint of metrology it would be helpful, if G could be related to quantities that have been measured precisely. Furthermore, the discussion in the previous section might be viewed as some - albeit very tentative - evidence, that G and the fundamental quantities that determine our

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[31]It requires quite a bit of faith to attribute a definite numerical value to a running quantity.

length and time-scales, such as α and m_e, should be related in one way or the other: In the previous section it has been remarked, that at high energies our definition of the meter, which is linked to the second via the speed of light, c, might change due to the running of the coupling constant(s). A change in the length scale will affect the maximum entropy of a space-time region bounded by a proper area A, unless there is a definite relation between the gravitational constant G and the fundamental quantities or processes that define our length or time scale at any particular energy, which allows us to calculate the change.

How can we define a time- or length scale at an arbitrary energy? In order to do this we need a physically realizable clock, which works (at least in principle) over the whole energy range from the Planck-energy to the low energies we encounter today. In order to define a time-interval as reference, we need a massive particle (time has no meaning for photons). This particle should be available throughout the whole energy range. From a practical point of view it should also be fundamental, meaning not composite, so that its properties and behavior can be "easily" calculated. The only particle which fulfils these conditions is the electron. At low energies we could construct a positronium clock. At high energies it might be more appropriate to use the compton wave-length of the electron as reference. In any case, the fundamental time or length interval determined from our electron based clock will depend on the energy scale, due to the running value of the fine-structure constant and the running electron mass. Unless there is a definite relationship between G, $\hbar$, α and m_e it will be difficult to determine the entropy of a self gravitating system in an unambiguous way.

How could such a relationship look like? The quantity $\hbar c/G$ has the dimension of mass-squared. In order to get the dimension right, we should start with

$$\hbar c/G = m_{Pl}^2 \propto m_e^2 \tag{88}$$

Unfortunately there is a great discrepancy between the Planck-mass and the electron-mass. Nevertheless, m_{Pl}/m_e is a dimensionless quantity, that might be related to the only other dimensionless fundamental quantity available at the low-energy scale, α. Therefore we make the following ansatz:

$$\ln\left(\kappa \frac{m_{Pl}}{m_e}\right) = \frac{x}{\alpha} \tag{89}$$

This ansatz is motivated by the renormalization group equations, according to which the coupling constants vary with energy with

$$\frac{1}{\alpha(E)} - \frac{1}{\alpha(E_0)} = \frac{b}{2\pi} \ln \frac{E}{E_0} \tag{90}$$

where b depends on the model.[32]

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[32] $b[U(1)] = -\frac{4}{3} N_g$ and $b[SU(N)] = \frac{11}{3} N - \frac{4}{3} N_g$, where N_g is the number of generations (three in the standard model).

What could be a reasonable value for x? There is some "evidence" for $x = 3/8$. In the simplest SU(5) GUT-theory the normalization of the electric charge operator with respect to the other operators requires this value. Furthermore, $3/8$ is the prediction for the Weinberg angle at the GUT-energy in minimal SU(5), i.e. the ratio of the electromagnetic coupling to the "true" unified coupling constant at the unification energy.

With $x = 3/8$ and setting κ temporarily to 1 equation (89) reads:

$$\ln\left(\frac{m_{Pl}}{m_e}\right) \approx \frac{3}{8\alpha} \tag{91}$$

If we plug in the experimentally determined values for the electron-mass, the fine-structure constant and the Planck-mass (which requires knowledge of G, $\hbar$ and c), we find a not too good agreement:

$$\ln\left(\frac{m_{Pl}}{m_e}\right) = 51.5279 \tag{92}$$

and

$$\frac{3}{8\alpha} = 51.3885 \tag{93}$$

However, there is still one free parameter in the ansatz of equation (89). Furthermore, the motivation for the search of a relation between G, $\hbar$, m_e and α was the observation, that the length-scale (or area scale) might change at high energies. Therefore the fundamental area r_0^2, which somehow "documents" this change, should enter into the above relation. For an electrically charged spin $1/2$ particle, such as the electron, the radius r_h of its boundary (or membrane) can be found in [5]:

$$\frac{r_h^2}{\hbar G} = \frac{\beta}{4} = \frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}} \tag{94}$$

Quite curiously, if we replace $m_e \rightarrow m_e/(\beta/4)$ the logarithms in equations (92, 93) are equal within the measurement errors[33]:

$$\ln\left(\frac{\beta}{4}\frac{m_{Pl}}{m_e}\right) = 51.3883 \approx \frac{3}{8\alpha} = 51.3885 \tag{95}$$

With $m_{Pl}^2 = \hbar c/G$ we can solve the above equation for G:

$$G = \frac{\hbar c}{m_e^2}\left(\frac{\alpha}{2} + \sqrt{\left(\frac{\alpha}{2}\right)^2 + \frac{3}{4}}\right)^2 e^{-\frac{3}{4\alpha}} \tag{96}$$

which gives the following "prediction" for G:

$$G = 6.670460 \cdot 10^{-11}\frac{m^3}{kg\,s^2} \tag{97}$$

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[33]The error is dominated by the uncertainty of m_{Pl} due to the large uncertainty of G

This is well within the errors of the value recommended by CODATA 1998 ($G = 6.673(10) \cdot 10^{-11} m^3/(kg\, s^2)$) and within 3σ of the "old" value $G = 6.6726(7) \cdot 10^{-11} m^3/(kg\, s^2)$, recommended by the International Council of Scientific Units 1986, which however was discarded by CODATA in 1998 because of difficulties to reproduce this result.

Keep in mind, however, that this section is pure numerology, which has not much to do with predictive science. As far as is known to the author, there is no example in the history of science, where numerology has guided us to any significant scientific result.

8 Fermionic weighting factors for the energy- and number-density

It is useful to cast the equations of the previous sections into a more familiar form. Usually the effective degrees of freedom of a gas of ultra-relativistic bosons and fermions are calculated by weighting the fermion degrees of freedom with a factor $w = 7/8$. This procedure allows us to apply the familiar Planck formula for the energy-density of a photon gas to the arbitrary case of a gas of ultra-relativistic bosons and fermions, simply by replacing the two photon degrees of freedom[34] in the Planck-formula with the effective degrees of freedom of the fermions and bosons.

The factor $7/8$ for each fermionic degree of freedom is nothing else than the ratio of $Z_{F,3}(u_F = 0)$ by $Z_{B,3}(u_B = 0)$. This ratio is relevant for the determination of the energy-density.[35] The weighting factor $7/8$ for the fermionic degrees of freedom is only correct, when $u_F = u_B = 0$, i.e. when the chemical potential of all particles is zero. For the holostar in thermodynamic equilibrium $u_F = 0$ is not possible, at least not in the simple ultra-relativistic model discussed here. u_F is always larger than 1.344 and depends on the ratio of the (unweighted) degrees of freedom of the ultra-relativistic bosons and fermions. Nonetheless, we can still use the standard Planck-formula for a photon gas by adhering to the following procedure:

- Determine the bosonic and fermionic particle degrees of freedom, f_B and f_F.

- Calculate u_F as a function of f_B/f_F from equation (56).

- Determine the fermionic weighting factors w_E and w_N for matter and anti-matter respectively. The weights only depend on u_F.

- Use w_E instead of $7/8$ and w_N instead of $3/4$ as the appropriate weighting factors for the fermionic degrees of freedom in order to determine the effective degrees of freedom of the whole gas.

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[34]Note, that in our convention for the counting of the degrees of freedom $g = 2$ corresponds to $f_B = 1$

[35]The ratio $Z_{F,2}(0)/Z_{B,2}(0) = 3/4$ is relevant for the number-density

If we denote by $\widetilde{f_E}$ the effective degrees of freedom required for the determination of the energy density and by $\widetilde{f_N}$ the effective degrees of freedom required for the number-density we get:

$$\widetilde{f_E} = 2f_B + (w_E + \overline{w_E})f_F \tag{98}$$

$$\widetilde{f_N} = 2f_B + (w_N + \overline{w_N})f_F \tag{99}$$

with

$$w_E = \frac{Z_{F,3}(u_F)}{Z_{B,3}(0)} \tag{100}$$

and

$$w_N = \frac{Z_{F,2}(u_F)}{Z_{B,2}(0)} \tag{101}$$

The weighting factors for the fermionic anti-matter $\overline{w_E}$ and $\overline{w_N}$ are given by the respective negative value of the fermionic chemical potential per temperature, i.e. $u_F \rightarrow -u_F$:

$$\overline{w_E} = w_E(-u_F) = \frac{Z_{F,3}(-u_F)}{Z_{B,3}(0)} \tag{102}$$

and

$$\overline{w_N} = w_N(-u_F) = \frac{Z_{F,2}(-u_F)}{Z_{B,2}(0)} \tag{103}$$

The quantity w_E gives the ratio of the total energy density of a single fermionic degree of freedom to a single bosonic degree of freedom (with zero chemical potential) in an arbitrary volume, i.e.

$$w_E = \frac{\delta E_F}{\delta E_B} \tag{104}$$

whereas w_N gives the ratio of the total number of fermions to bosons (for a single bosonic and fermionic degree of freedom) in an arbitrary volume:

$$w_N = \frac{\delta N_F}{\delta N_B} \tag{105}$$

The respective ratios of the energy- and number-densities of any fermion with respect to its anti-particle are then given by $w_E/\overline{w_E}$ and $w_N/\overline{w_N}$

The mean energy per particle is proportional to the temperature. However, the constant of proportionality is different for fermions and bosons:

$$\frac{\delta E_B}{\delta N_B} = \varepsilon_B T \tag{106}$$

and

$$\frac{\delta E_F}{\delta N_F} = \varepsilon_F T \tag{107}$$

with

$$\varepsilon_B = \frac{\widetilde{f_N}}{\widetilde{f_E}} \sigma \tag{108}$$

but

$$\varepsilon_F = \frac{w_E}{w_N} \frac{\widetilde{f_N}}{\widetilde{f_E}} \sigma \tag{109}$$

The ratio of the mean energy of a single fermionic degree of freedom to a single bosonic degree of freedom is given by:

$$\frac{\varepsilon_F}{\varepsilon_B} = \frac{w_E}{w_N} \tag{110}$$

The above dependencies guarantee, that the mean energy per particle per temperature ε (for all particles, i.e. including all fermions and bosons) is equal to the mean entropy per particle σ (again for all particles):

$$\frac{\delta E}{\delta N} = \varepsilon T = \sigma T \tag{111}$$

with

$$\varepsilon = \frac{f_F(N_F \varepsilon_F + \overline{N_F \varepsilon_F}) + 2 f_B N_B \varepsilon_B}{f_F(N_F + \overline{N_F}) + 2 f_B N_B} = \varepsilon_B \frac{\widetilde{f_E}}{\widetilde{f_N}} = \sigma \tag{112}$$

Note that the relation $\varepsilon = \sigma$, which can be seen as the basic thermodynamical characteristic of the holostar, is only true for the mean energy and entropy of all particles, but not for the individual particle species, for which the relations (108, 109) hold. These relations in general imply $\varepsilon_B \neq \sigma_B$ and $\varepsilon_F \neq \sigma_F$, unless the number of bosonic degrees of freedom is zero.

The mean entropy per particle σ can be calculated from the mean entropy per fermion and boson:

$$\sigma = \frac{f_F(\sigma_F w_N + \overline{\sigma_F w_N}) + 2 f_B \sigma_B}{\widetilde{f_N}} \tag{113}$$

With all of the above definitions equation (52) reads as follows[36]:

$$4\pi\beta = \frac{\widetilde{f_E}}{15 \cdot 16} \tag{114}$$

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[36]In the scale-invariant reformulation of the Hawking entropy in section 6 we have to use equation (81), which then is given by $4\pi\beta = \frac{\widetilde{f_E}}{15 \cdot 16} \left(\frac{\beta}{4}\right)^4$

f_B/f_F	u_F	σ	σ_F	$\overline{\sigma_F}$	w_N	$\overline{w_N}$	w_E	$\overline{w_E}$
0	1.34416	3.3792	3.2006	5.4052	2.3865	0.2103	3.0114	0.2372
1/100	1.35046	3.3758	3.1966	5.4112	2.3986	0.2091	3.0282	0.2357
1/10	1.40496	3.3489	3.1615	5.4626	2.5054	0.1983	3.1766	0.2234
1/5	1.46148	3.3248	3.1256	5.5162	2.6204	0.1877	3.3375	0.2113
1/3	1.53124	3.2993	3.0820	5.5824	2.7685	0.1753	3.5462	0.1972
1/2	1.61097	3.2750	3.0330	5.6584	2.9464	0.1622	3.7991	0.1823
1	1.81380	3.2299	2.9126	5.8529	3.4424	0.1330	4.5175	0.1492
2	2.12023	3.1902	2.7421	6.1493	4.3213	0.0984	5.8340	0.1101
3	2.35325	3.1735	2.6213	6.3764	5.1053	0.0782	7.0515	0.0873
5	2.70549	3.1607	2.4524	6.7219	6.5032	0.0552	9.3134	0.0615
8	3.08811	3.1568	2.2865	7.0994	8.3468	0.0377	12.456	0.0420
10	3.28970	3.1575	2.2059	7.2989	9.4688	0.0309	14.448	0.0344
100	6.21570	3.2236	1.4151	10.216	41.804	0.0017	88.571	0.0019
1000	11.3315	3.3308	0.8378	15.332	217.25	10^{-5}	734.05	10^{-5}
3000	14.9899	3.3793	0.6436	18.990	487.51	$3 \cdot 10^{-7}$	2116.2	$3 \cdot 10^{-7}$

Table 1: Thermodynamic parameters for an ultra-relativistic gas of fermions and bosons, compiled for selected ratios of bosonic to fermionic degrees of freedom f_B/f_F. u_F is the dimensionless chemical potential per temperature of the fermions. Anti-fermions have the opposite value. The chemical potential of the bosons is zero. σ is the mean entropy per particle, σ_F is the entropy per fermion. w_E and w_N are the weighting factors for the energy- and number-densities with respect to a bosonic degree of freedom. Barred quantities refer to anti-fermions.

The effective degrees of freedom determined by the above procedure can be plugged into the well known equations for the energy-density, the entropy density and the number-density of a photon gas. For example, the energy-density is now given as:

$$\rho = \frac{1}{8\pi r^2} = \widetilde{f_E} \frac{\pi^2}{30\hbar^3} T^4 \tag{115}$$

In Table 1 the relevant thermodynamic quantities are compiled as a function of the ratio f_B/f_F of bosonic to fermionic degrees of freedom. All the quantities in the table are normalized to the thermodynamic quantities of an ideal relativistic boson gas. Recall, that for a relativistic boson gas $\sigma_B = 2\pi^4/(45\zeta(3)) = 3.60157$ and $\varepsilon_B = \pi^4/(30\zeta(3)) = 2.70118$.

We find, that the fermions within the holostar have much higher weights for the energy- and number-densities than the ordinary weights of 7/8 or 3/4, whereas the anti-fermions have much lower weights. For example, when $f_F = f_B$, the weighting factor for the fermionic energy-density, 7/8, must be replaced by 4.5 and that for the anti-fermion with 0.15.

An interesting case is $f_B = 8f_F$, i.e. when there are 8 bosons per fermion. For this case the entropy per particle σ (and the energy per particle) within

f_B/f_F	u_F	σ	σ_F	σ_B	w_E	w_N	w_{EB}	w_{NB}
0	1.11721	3.3516	3.3516	5.0269	2.4620	1.9842	0.3089	0.2846
1/10	1.14848	3.3534	3.3304	5.0613	2.5318	2.0359	0.2992	0.2754
1/3	1.21268	3.3581	3.2872	5.1314	2.6808	2.1457	0.2802	0.2575
1	1.35321	3.3717	3.1948	5.2835	3.0355	2.4039	0.2428	0.2225
3	1.61169	3.4059	3.0325	5.5589	3.8015	2.9480	0.1868	0.1704
10	2.03254	3.4787	2.7895	5.9986	5.4263	4.0529	0.1221	0.1108
100	3.12422	3.7293	2.2717	7.1131	12.795	8.5398	0.0407	0.0368
1000	4.45250	4.1130	1.8183	8.4496	31.935	18.341	0.0108	0.0097

Table 2: Thermodynamic parameters for the "abnormal supersymmetric phase" of an ultra-relativistic gas consisting of fermions and bosons, compiled for selected ratios of the bosonic to fermionic degrees of freedom f_B/f_F. u_F is the chemical potential per temperature of the fermions, the bosons have the opposite value. s is the mean entropy per particle, σ_F and σ_B are the entropies per fermion and boson, respectively. w_E and w_N are the weighting factors for the energy- and number-densities of the fermions. The weighting factors of the bosons are denoted by subscript B.

the holostar is minimized. Note that $\sigma = 3.1568$ is only slightly larger than π: $\sigma/\pi = 1.00485$.

There is a curious modification to the thermodynamic model, which might be of some interest. If we formally assume, that the fermions have no anti-particles, we can give the bosons a negative chemical potential exactly opposite to that of the fermions, $\mu_F + \mu_B = 0$, and still retain a symmetric description. In a formal sense the bosonic partner particle of any fermion, carrying the opposite chemical potential, might be considered as the anti-particle of its fermionic counterpart. Let's call this peculiar matter phase the "abnormal supersymmetric phase". We can construct a table similar to Table 1. The respective weighting factors for the bosonic energy- and number-densities are given by:

$$w_{EB} = \frac{Z_{B,3}(-u_F)}{Z_{B,3}(0)} \tag{116}$$

$$w_{NB} = \frac{Z_{B,2}(-u_F)}{Z_{B,2}(0)} \tag{117}$$

The essential thermodynamic parameters of the "abnormal" supersymmetric phase are compiled in Table 2

It is a curious numerical coincidence, that for the "abnormal supersymmetric phase" with identical bosonic and fermionic particle degrees of freedom the ratio

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of the entropy per boson σ_B to the entropy per fermion σ_F in the holostar is almost equal to the ratio predicted for the respective areas of a single spin-1 spin-network state (=boson?) to a single spin 1/2 spin-network state (=fermion?) in loop quantum gravity (LQG):

$$\left(\frac{\sigma_B}{\sigma_F}\right)_{holo} \simeq 1.654 \tag{118}$$

whereas:

$$\left(\frac{\sigma_B}{\sigma_F}\right)_{LQG} = \sqrt{\frac{j_B(j_B+1)}{j_F(j_F+1)}} = \sqrt{\frac{8}{3}} \simeq 1.633 \tag{119}$$

Whether this finding is significant, is hard to tell. Although there appears to be a connection between the ultra-relativistic particles of the holostar solution with the links of a loop quantum gravity (LQG) spin-network state, it is yet too early to draw any definite conclusions. See [5] for a more detailed discussion on the possible connection between LQG and the classical holographic solution.

There is another interesting observation. If we compare the first line in Table 1, i.e. the case of a "normal" gas consisting exclusively out of ultra-relativistic fermions and anti-fermions, with the $f_F = f_B$ line in Table 2, i.e. the "abnormal supersymmetric-phase" with equal fermionic and bosonic degrees of freedom, we find that the thermodynamic properties of both phases are very similar. The chemical potentials per temperature are $u = 1.344$ in the first case and $u = 1.353$ in the second case. The mean entropy per particle is $\sigma = 3.379$ in the first case and $\sigma = 3.372$ in the second case. If we compare the other quantities in the table, i.e. the entropies of the fermions, the entropies of the anti-fermions/bosons, the weighting factors for the fermions, the weighting factors for the anti-fermions/bosons, we also find, that all of these quantities are very similar. Therefore, from a purely thermodynamic point of view, a "normal" ultra-relativistic gas-phase consisting only out fermions and anti-fermions has nearly identical properties to the "abnormal supersymmetric" gas-phase consisting out of an equal number of fermions and bosons, with the interpretation that the bosons have "disguised" themselves as the anti-particles of the fermions. So in a strictly formal sense one could say, that at ultra-high temperatures a gas consisting exclusively out of fermions and their anti-particles becomes more or less indistinguishable from a gas consisting out of equal numbers of fermions and bosons.

9 On the matter-antimatter asymmetry in curved space-times

In the previous sections we have seen, that there is only a solution to the thermodynamic constraint equation $F_S(u_F) = F_E(u_F)$, when we have at least one ultra-relativistic fermionic species present. No matter what the specifics of the thermodynamic model are, the ultra-relativistic fermions are required to have a

non-zero chemical potential, which is significantly higher than the local radiation temperature. We can loosely interpret this finding such, that we need the degeneracy pressure of at least one ultra-relativistic fermion in order to stabilize the self gravitating object, so that it doesn't collapse under its own gravity to a singularity.

Table 1 describes the characteristic properties of an ideal gas of ultra-relativistic fermions and bosons, which are in thermal equilibrium with each other and their anti-particles. The non-zero chemical potential of the fermions induces an asymmetry in the relative number-densities of a fermionic particle and its anti-particle. From this asymmetry different values for the entropy per particle/anti-particle and the ratio of the energy-densities and number-densities arise.

The asymmetry is smallest, when at a certain spatial position within the holostar there are only fermions and no bosons. For this situation the chemical potential per temperature u_F attains its minimum value of $u_F \simeq 1.34416$.

If we increase the number of bosonic species with respect to the fermions, the matter-antimatter asymmetry in the fermions becomes higher with increasing u_F, as can be seen from comparing the first and last columns of the Table 1. For $f_B = 0$ we have $w_E \simeq 3.01$ and $\overline{w_E} \simeq 0.24$, so that the ratio of the energy-densities of matter vs. anti-matter in any proper volume (where the fermions still are ultra-relativistic) is: $\eta_E = 12.6967$. The number-densities have roughly the same ratio: $\eta_N = \overline{w_N}/w_N = 11.3453$. When the number of fermionic and bosonic species is equal, we find that $\eta_E = 30.2894$ and $\eta_N = 25.8847$, i.e. a significantly higher asymmetry.

The holsostar's interior structure, or rather the condition $\sigma = \epsilon/T$, induces a natural asymmetry between the fermionic matter and antimatter in thermodynamic equilibrium in the spatial holostar metric. Only the fermions are effected in such a way. For the bosons there is no such shift, because they cannot have a non-zero chemical potential, at least as long as they are ultra-relativistic.

If the interior temperature falls below the mass-threshold of a particular fermionic species, fermions and anti-fermions will annihilate. Due to the large asymmetry above the threshold most fermions will survive the annihilation process. The ratio of the number of surviving fermions with respect to the total number of fermions and anti-fermions above the threshold is:

$$\eta = \frac{\eta_N - \overline{\eta_N}}{\eta_N + \overline{\eta_N}}$$

This ratio attains its minimum value in the case where there are only fermions, no bosons, i.e. $f_B/f_F = 0$, where $\eta \approx 84\%$. The ratio can be significantly higher, if the number of bosonic species is higher.

In [7] it has been shown, that the holostar has some potential to serve as an alternative model for the universe. If this truly turns out to be the case, the thermodynamic properties of the holostar solution naturally explain the matter-antimatter asymmetry in the universe: When during the expansion the temperature falls below the rest-mass of a particular fermionic species, for example the baryons (or quarks, electrons), the fermions will annihilate with their anti-partners. Due to the curvature-induced matter-antimatter asymmetry there will

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be at least a factor of 11 more particles than antiparticles, so that the mutual annihilation conserves most of the energy-density within any particular fermionic species, at least 84 %. This figure might be significantly higher, when there are many more bosonic than fermionic species. However, in the Standard Model of particle physics the situation is rather the other way around: We have more fermions than bosons. Whenever $f_F \geq f_B$, i.e. the number of fermionic degrees of freedom is larger or equal than the bosonic degrees of freedom, the percentage of fermions surviving the annihilation process with respect to the total number of fermions before the annihilation started, is only very moderately dependent on the ratio f_B/f_F. It ranges from 84% at $f_B = 0$ over 88% at $f_F = 3f_B$ to 92.5% at $f_F = f_B$.

This effect also explains, why r_0^2, which has been shown to depend linearly on the effective ultra-relativistic degrees of freedom, has a nearly universal value, although the number of relativistic particle degrees of freedom is dramatically reduced during the expansion: Due to the large matter-antimatter asymmetry the energy-density in a particular particle species is nearly conserved, when the species "freezes out", so that the effective number of the degrees of freedom at the transition doesn't change significantly.

This observation is significant in two respects:

First it allows us to interpret the energy density in the universe as we find it today, as a fairly good indicator of the number of degrees of freedom at very high energies, where all particle species are expected to be ultra-relativistic. In fact, the "effective" number of degrees of freedom, determined via the energy-density of the matter today, is expected just to slightly underestimate the total number of degrees of freedom at the Planck scale. In the worst case the observed total energy density would be roughly 85% of the energy-density at a temperature, where all of the particles were relativistic. We will see in the next section, that this appears to be actually the case.

Second, it gives a good a-posteriori justification for our very early assumption, that the fundamental area r_0^2 is a (nearly) universal quantity, constant whenever the universe doesn't undergo a phase-transition, and which "runs" only moderately with the energy-scale via the coupling-constant(s), whose values are related to the particle degrees of freedom that are available at a given energy.

Whereas the interpretation of r_0^2 as a running area scale, only slightly dependent on the energy-scale, can be considered to be backed by both observational and theoretical insight, the particular form of r_0^2 as proposed in equation (78) has not such a sound justification, at least at high energies. Our understanding of the holostar solution and particle physics in curved space-times will improve dramatically, if the apparent correspondence between the fundamental length- and energy-scales, the number of particle degrees of freedom and the running of the coupling constant(s) can be made more definite.

10 An estimate for the number of degrees of freedom at the Planck scale

With the discussion beforehand and the tables given in section 8 one can estimate the number of degrees of freedom at the Planck energy, where all particle degrees of freedom are expected to be ultra-relativistic.

I will only discuss the "normal" matter phase, whose properties are given by Table 1. With f let us denote the total number of degrees of freedom, i.e. $f = 2(f_B + f_F)$. If supersymmetry is a true symmetry of nature at high energies, we should expect the bosonic and fermionic degrees of freedom to be equal at the Planck energy, which gives: $f_B = f_F = f/4$.

From equation (52) we find:

$$\widetilde{f_E} = \frac{f}{2}(1 + \frac{w_E + \overline{w_E}}{2}) = 2^6 \cdot 3 \cdot 5 \cdot \pi \cdot \beta \tag{120}$$

and therefore[37]:

$$f = \frac{2^7 \cdot 3 \cdot 5 \cdot \pi \cdot \beta}{(1 + \frac{w_E + \overline{w_E}}{2})} \tag{121}$$

For $f_F = f_B$ one can show (see [9]):

$$\frac{w_E + \overline{w_E}}{2} = \frac{7}{3} \tag{122}$$

so that:

$$f = 2^4 \cdot 3^2 \cdot 4\pi\beta \tag{123}$$

In order to determine f we need to know $\beta = r_0^2/\hbar$ at the Planck scale. In [5] arguments were given, that $\beta/4 \approx \sigma/\pi \simeq 1$ at the Planck energy[38]. A more conservative estimate will place β in the following range:

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[37]If the Hawking entropy area law were modified according to the discussion in section 6, the above expression must be multiplied with $(4/\beta)^4$:

$$f = \frac{2^7 \cdot 3 \cdot 5 \cdot \pi \cdot \beta}{(1 + \frac{w_E + \overline{w_E}}{2})} \left(\frac{4}{\beta}\right)^4$$

[38]It is suggestive to set $\beta = 4$ at the Planck-energy, which at the same time will set $\alpha = 1/4$. This would make the prediction $f = 2^6 3^2 4\pi$. The factor of π is somewhat disappointing. One would expect an integer number, at least if particles were the truly fundamental building blocks of nature. The basic building blocks of the holostar, however, appear to be rather strings and membranes. Therefore it is conceivable, that the particle degrees of freedom must be regarded as an effective description. For an effective description non-integer values for f are not uncommon. The full number of degrees of freedom at or above the Planck-scale, including the "stringy" degrees of freedom, is expected to be higher and integer. Only a unified theory of quantum gravity, which most likely will be based on string-theory, will be able to tell us, what the full spectrum of the basic building blocks of nature, i.e, their interior structure and their relative abundances, is going to be.

$$\sqrt{\frac{3}{4}} \leq \frac{\beta}{4} \leq \frac{\sigma}{\pi} \tag{124}$$

For the above range we find from equation (123):

$$6269 < f < 7442 \tag{125}$$

Note that the ratio of the lower to the higher number in this range is very close to of 84 %. This is almost exactly the ratio one expects for the effective degrees of freedom at low vs. high energies due to the matter-antimatter asymmetry (see section 9). In fact, $6269/7442 = 0.842$.

For $\beta = 4$ we have $f \approx 7238$. These are all quite large numbers compared to the number of particle degrees of freedom of the Standard Model. On the other hand it has been speculated, that at exceedingly higher energies more and more new particles will show up, and that this process might continue indefinitely. Although the numbers stated here cannot yet be regarded as an accurate prediction, one can interpret the above result such, that the number of fundamental particles is finite and is expected to lie not too far outside range given in equation (125). Thus we have a good chance to discover a unified description of nature, encompassing all known forces and matter states.

The lower value in the range given by equation (125), 6270, is quite close to the experimental estimate of f, which can be obtained from equation (115), using the temperature of the microwave background radiation and the total matter density as input. With $T_{CMBR} = 2.725\,K$ and ρ determined from the recent WMAP data [3] we find the following experimental estimate for the effective degrees of freedom at the low energy scale:

$$f = \frac{60}{\pi^2} \frac{\rho}{T^4} \frac{\hbar^3}{(1 + \frac{w_E + \overline{w_E}}{2})} = \frac{18}{\pi^2} \frac{\rho}{T^4} \hbar^3 \approx 6366 \tag{126}$$

Therefore the assumption, that $r_0^2 = \beta\hbar$ is a nearly universal area scale, only depending on the effective degrees of freedom at a given energy scale via the relevant coupling constants, has some observational justification.

11 Does the holographic solution conserve the ratios of the energy-densities of the fundamental particle species?

Quite interestingly $f/4$ of equation (125) is not too far from the ratio of proton to electron mass ($m_p/m_e = 1836.15$). As the electron has four states (according to the Dirac-equation), the ratio of proton to electron-mass is roughly equal to the ratio of the *total* number of particle degrees of freedom in the holostar-solution to the *four* degrees of freedom of the electron.

At first sight this looks like quite a coincidence. However, there is another curious coincidence in our universe today: The energy-density of the microwave-background radiation is roughly equal to the energy density of the electrons within a factor of 2.5 (if we assume that there is no dark matter). Both electrons and photons are fundamental particles. If the near equivalence of photon energy density to electron energy density is not just a spurious feature of the universe in its present state[39], one wonders, whether the universe might be constructed such, that the ratio of the energy-densities of the different fundamental particle species should remain approximately constant throughout its evolution, endowing every fundamental degree of freedom with a well-defined energy-density.

In [7] it was shown, that for the geodesic motion of massless and massive particles, and for some particular cases of non-geodesic motion of massive particles, the ratios of the energy-densities of the particles are conserved in the interior holostar space-time. For these particular cases the conjecture has the status of being proved.

There are indications that such a conjecture is also valid with respect to the distribution of electromagnetic (and rotational) energy in the interior of a charged and/or rotating holostar: In the model for a charged holostar discussed in [5] the ratio of electromagnetic energy density $\rho_{em} \propto 1/r^2$ to the total energy density $\rho = 1/(8\pi r^2)$ is constant throughout the whole holostar's interior and is related to the dimensionless ratio of the holostar's exterior conserved charge Q^2 and boundary area A via $\rho_{em}/\rho_{tot} = Q^2/r_h^2 = 4\pi Q^2/A = const.$

If one elevates the above stated conjecture to the status of a general principle, one is more or less forced to regard the nucleon, which quite definitely is not a fundamental particle, as some sort of "dump" for the frozen out fundamental degrees of freedom at the Planck scale, which have become "locked away" in the interior structure of the nucleon. According to the discussion in section 9 at least 84 % of the energy-density in the ultra-relativistic particles at the Planck-scale must be preserved in the subsequent evolution, due to the profound matter-antimatter asymmetry at ultra-relativistic energies. Using Occams razor (which allows us to ignore the unsettled issues of cold dark matter or dark energy), where else than into the nucleon could the energy density of these frozen out degrees of freedom have gone to?

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[39]In the standard cosmological model the energy-density of the photons with respect to the electrons changes with time, as the photons are "red-shifted" away and the ratio of photons to electrons is expected to remain constant. However, in a non-homogeneous universe with significant pressure it is not altogether clear, if the number ratio of zero rest-mass particles to massive particles in the cosmic fluid should remain constant. In fact, in the holostar universe one can show (see [7]) that this ratio develops proportional to $\overline{n}_\gamma/\overline{n}_e \propto m_e/T$ in the frame of the co-moving observer, so that the red-shift of the photons is compensated by their higher number-densities, keeping the ratio of the respective energy densities constant.

11.1 An estimate for the ratio of the energy-density of photons to electrons

If we take the conjecture seriously, that the universe preserves the ratios of the energy-densities of its fundamental constituents during its evolution, we should be able to estimate the ratio of the energy-densities of electrons to photons by a thermodynamic argument. When the temperature in the holographic universe reaches the electron-mass threshold, we have $f_F = 5$ ultra-relativistic fermionic particle species around: 3 flavors of neutrinos and two helicity-states for the electrons. If right-handed neutrinos and left-handed anti-neutrinos exist, we have to add three more degrees of freedom, i.e. $f_F = 8$. The photons are counted as $f_B = 1$, so $f_F/f_B \approx 5-8$. In this range, the relative number-density of the electrons with respect to the photons is given by $w_N \approx 2.6$, according to Table 1. The number-density of the positrons is roughly $\overline{w_N} \approx 0.19$ that of the photons. The ratios of the energy-densities of electrons *and* positrons to photons is roughly $w_E + \overline{w_E} \approx 3.5$. After the annihilation of the positrons roughly 86 % of the original energy-density will "survive" in the left-over electrons. The 14% gone into the annihilation is distributed among all left over particles, i.e. neutrinos, photons and electrons.[40] The final result is, that the energy-density of the left-over electrons with respect to the energy-density of the photons should lie in the range

$$\frac{e_e}{e_\gamma} \approx 2.0 - 2.8$$

The lower value refers to the case, when the neutrinos are already decoupled, so that the photons get the full share of the annihilation energy (14%). When the neutrinos are not yet decoupled, they will take a large fraction of the 14% annihilation energy, leading to the higher value.

Remarkably the observationally determined estimate of the ratio of the energy-density of the electrons to the photons is quite close to the above figure. When we estimate the electron contribution to the total matter-density as determined by WMAP, under the assumption that all of the matter is baryonic, and compare this to the known energy-density of the CMBR, we get a ratio of 2.5, as was shown in [7].

11.2 An estimate for the ratio of proton to electron mass

With a similar argument one can estimate the proton to electron mass ratio. We have seen that at ultra-high temperatures the total number of particle degrees of freedom in the holostar-solution is given by

$$f = 2^4 3^2 4\pi\beta$$

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[40]In principle the nucleons could also participate in the energy-transfer. As long as the nucleons are much heavier than the electrons at the electron-mass threshold, the energy-transfer to the nucleons will be small.

At low temperatures the only fundamental particle degrees of freedom left are the electrons ($f_e = 4$), the neutrinos ($f_\nu = 3 \cdot 2$) and the photons ($f_\gamma = 2$). The number of "missing" degrees of freedom is given by

$$\Delta f = 2^4 3^2 4\pi\beta - 12$$

In a spherically symmetric space-time mass-energy is conserved. The mass-energy of the missing particle degrees of freedom must show up somewhere. At low energies the natural candidate for the missing degrees of freedom is the lightest surviving compound particle, the proton. Therefore we can estimate the energy-density of protons to electrons as follows

$$\frac{e_p}{e_e} \approx \frac{\Delta f}{4} = 2^2 3^2 4\pi\beta - 3$$

However, this is the ratio of the energy-densities when the electrons are still relativistic. When the electrons and positrons finally annihilate, we have estimated in the previous section that only roughly 84 % of their original energy-density finds its way to the surviving electrons, so that the ratios of the energy-densities after the annihilation of the positrons has to be corrected by this factor. At low temperatures the ratios of the energy-densities are nothing else than the ratios of the respective rest-masses, so that the ratio of proton to electron mass can be estimated as:

$$\frac{m_p}{m_e} \approx \frac{2^2 3^2 4\pi\beta - 3}{0.84} \tag{127}$$

If we set β to the value determined in [5], i.e. $\beta \simeq 4\sqrt{3/4}$ we find:

$$\frac{m_p}{m_e} \approx 1862 \tag{128}$$

This is quite close to the actual value $m_p/m_e = 1836.15$. Alternatively one could use equation (127) to get another experimental estimate for the fundamental area $r_0^2 = \beta\hbar$.

In a more sophisticated treatment one would have to take into account the bosonic and fermionic degrees of freedom separately. Furthermore the neutron and the different chemical potentials of neutrons and protons cannot be neglected.[41] However, with our limited understanding of the holographic solution at the current time it does not seem appropriate to stretch an order of magnitude estimate far beyond its already limited range of credibility.

12   Supersymmetry

We have seen in the previous section, that the supersymmetric case is special in the sense, that the ratio of the energy-densities of fermions (including the

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[41]Assuming that the chemical potentials of u and d quarks are equal, and assuming that the chemical potentials of the constituent quarks can be summed up, the chemical potential of the neutron ($\overline{dd}u$) will be *opposite* to the chemical potential of the proton ($uu\overline{d}$).

anti-fermions) to bosons takes on an integer value. The chemical potential per temperature of the fermions in the supersymmetric case is given by

$$u_{ss} = \frac{\pi}{\sqrt{3}} \simeq 1.8138$$

For this value of u we find:

$$\frac{E_F + \overline{E_F}}{2E_B} = \frac{Z_{F,3}(u_{ss}) + Z_{F,3}(-u_{ss})}{2Z_{B,3}(0)} = \frac{w_E + \overline{w_E}}{2} = \frac{7}{3} \qquad (129)$$

A similar relation holds for the entropy-densities:

$$\frac{S_F + \overline{S_F}}{2S_B} = \frac{w_N \sigma_F + \overline{w_N \sigma_F}}{2\sigma_B} = \frac{3}{2} \qquad (130)$$

With the above result one can relate u_{ss} to the thermodynamic parameters of the system:

$$u_{ss} = \frac{20}{9} \frac{e_B}{\Delta n_F\, T} = \frac{\pi^4}{27\zeta(3)} \frac{n_\gamma}{\Delta n_F} \simeq 3.0013 \frac{n_\gamma}{\Delta n_F} \qquad (131)$$

Δn_F is the fermion number density, i.e. the difference of the number of fermions minus anti-fermions per unit volume, e_B is the energy density of a single bosonic degree of freedom and n_γ is the number-density for a photon gas according to the Planck distribution (g=2). We find that in the super-symmetric phase the fermion number-density Δn_F is roughly a factor of 1.66 higher than the boson number density n_γ.

Not quite unexpectedly equations (129, 130) guarantee, that the free energy comes out zero, although the thermodynamic relations for an ultra-relativistic photon gas imply that its free energy-density is negative $f_B = e_B - s_B T = -e_B/3$, due to the well known relation between energy- and entropy-density for a photon gas:

$$s_B T = \frac{4}{3} e_B$$

If the free energy shall be zero $sT/e = 1$ is required. We get this by pairing any two bosonic degrees of freedom with a fermionic particle anti-particle pair. According to equation (130) the total entropy density is enhanced by a factor 5/2 by this pairing

$$s = s_B(1 + \frac{3}{2})$$

whereas the total energy density is multiplied by a factor 10/3

$$e = e_B(1 + \frac{7}{3})$$

The relation between the total energy- and entropy-density can be expressed in terms of the relation for a photon gas:

$$\frac{sT}{e} = \frac{\frac{5}{2}s_B T}{\frac{10}{3}e_B} = \frac{3}{4}\frac{s_B T}{e_B} = 1$$

13 Thermodynamics of the membrane

The simple model of an ultra-relativistic fermion and boson gas, subject to the interior spherically symmetric metric $g_{rr} = r/r_0$, reproduces the Hawking-entropy and -temperature, therefore giving a microscopic statistical explanation for the origin of the Hawking-entropy and -temperature, which fits well into the theoretical framework that has been developed for black holes over the last decades.

So far only the physics of the holostar's interior has been discussed. At least from the viewpoint of an exterior observer the properties of the membrane cannot be neglected. The surface pressure of the holostar's membrane carries a stress-energy equal to the holostar's gravitating mass. Furthermore, the membrane might substantially contribute to the entropy.

From the point of view of string-theory the properties of the membrane are quite easily explained. The interior strings are attached to the boundary membrane. Each string segment occupies a surface patch of of exactly one Planck area. The membrane has similar properties to that of a 2D-brane in string-theory. It has surface-pressure, but no interior mass-energy.

In this section I will try to interpret the properties of the membrane in terms of particles. It might turn out, that this is not the correct approach, and that the final answer has to be sought purely in the context of string theory. Yet the particle interpretation allows us to give some fairly self-consistent and (apparently) sensible answer to the questions which have been skipped in the previous sections, i.e. what contribution the membrane can or will make to the gravitating mass and the entropy of the holostar in the framework of the thermodynamic model that has been developed in the previous sections. Keep in mind that this section is somewhat speculative, as not much is known about self-gravitating matter-states which are effectively confined to a two-dimensional surface of spherical topology.

According to the holostar-equations the membrane has a large surface pressure. This property would be difficult to explain, if the membrane consisted only of weakly or non interacting particles.[42] The "forces" holding the membrane together must be strongly attractive and presumably long-ranged. Therefore photons or fermions seem not very well suited candidates in order to explain the properties of the membrane. Presumably the membrane consists of a gas or fluid of strongly interacting bosons.

A natural candidate for such a boson is the graviton. If the number of bosons in the membrane is comparable to the number of fermions inside the

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[42] In fact, the zero energy-density in the membrane suggests, that the membrane does not consist out of particles at all, and that it is a pure string structure.

holostar[43] the bosons will be very close to each other. Their mean separation will be of the order of the Planck-length. At such small distances gravity will have become a very strong force. Therefore mutual interactions of gravitons at close range could provide the "glue" holding the membrane together. Note also, that gravitons, being spin-2 particles, have the same transformation properties as the patches of a two-dimensional surface.

The radial metric-coefficient g_{rr} of the holostar space-time attains its largest value at the position of the membrane. The effective potential for the motion of massive and massless particles has a global minimum at the position of the membrane. Therefore the movement of particles in the radial direction is extremely "inhibited", whereas movement in the tangential direction, i.e. within the membrane itself, can be considered to take place essentially unhindered. For large holostars, the local movement of any particle in the vicinity of the membrane will be effectively constrained to its two-dimensional surface area.

The membrane is the minimum of the effective potential of the holostar. If we assume some weak, but non-zero interaction (friction) between the particles in the vicinity of the membrane, particles will collect in the membrane as the location of minimum energy. Whereas bosons have no problem to occupy the same volume, fermions are subject to the exclusion principle. A membrane which consists of a large number of particles separated by a proper distance of roughly Planck-size will quite likely contain a vast number of bosons, but essentially no fermions.

The total number of particles of an ideal gas of bosons moving freely in a two-dimensional surface of proper area A is given by:

$$N = N_0 + \frac{f}{2\pi\hbar^2}T^2 A Z_{B,1}(u_B) \qquad (132)$$

N_0 is the number of bosons condensed into the ground state, f is the number of degrees of freedom of the bosons in the membrane (for gravitons: 2), T is the temperature at the membrane and $Z_{B,1}$ is one of the integrals defined in section 4.

AT^2 is proportional to r_h and $Z_{B,1}(u_B)$ attains its maximum value for $u_B = 0$. Under the assumption that the total number of particles, N, in the membrane is proportional to r_h^2, and disregarding the ground state occupation N_0, the left side of equation (132) will grow much faster than the right side. At the Bose-temperature the total number of particles, N, will exceed the maximum value possible for the second term on the right side of equation (132). At this point the occupation of the ground state, N_0, must become macroscopic.

The transition temperature T_B from microscopic ground state occupation ($N_0 \sim 0$) to macroscopic occupation can be calculated from equation (132) by setting $N_0 = u_B = 0$. One finds:

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[43]Supersymmetry suggests a direct correspondence between fermions and bosons. If supersymmetry is relevant for large holostars, the assumption that the number of particles comprising the interior of the holostar (presumably dominated by fermions) should be proportional to the number of particles in the membrane (possibly dominated by bosons) doesn't seem too far fetched.

$$T_B^2 = \frac{N}{A}\frac{2\pi\hbar^2}{fZ_{B,1}(0)} = \frac{3\hbar}{f\pi\sigma} \approx 0.3\frac{\hbar}{f} \tag{133}$$

where $A = 4\sigma N\hbar$ was used, assuming that the number of bosons in the membrane is equal to the number of the holostar's interior particles. The Bose-temperature of the two-dimensional membrane is independent of r_h and of order of the Planck-temperature $T_{Pl} = \sqrt{\hbar}$. For reasonable values of f_B and f_F the numerical factor in (133) will lie in the interval from 0.285 and 0.303. With two degrees of freedom for the gravitons one finds that the Bose temperature of the membrane is roughly one third of the Planck temperature. The mean energy of a boson constrained to a two-dimensional surface is $E = (12\zeta(3)/\pi^2)\,T \approx 4.4T$. Therefore, at the Bose-temperature the mean energy of the bosons in the membrane slightly exceeds the Planck energy.

For large holostars the local temperature of the membrane will be far less then its Bose-temperature, due to the $1/\sqrt{r}$-dependence of the local temperature. Even for a holostar of Planck-size Bose condensation of the membrane is likely.[44] With the possible exception of very small holostars, all of the bosons comprising the membrane will be condensed into the ground state. Therefore the membrane will not contribute to the entropy.

On the other hand, the membrane's contribution to the energy cannot be neglected. The ground state energy for a single boson will be somewhat larger than the energy of a standing wave with a wavelength comparable to the proper circumference of the holostar:

$$E_0 \simeq \frac{\hbar}{2\pi r_h}\xi_0 \tag{134}$$

$\xi_0 \approx 3$ is the constant for the lowest vibrational mode of the membrane. Its exact value can be determined by solving the equations for a vibrating spherical membrane.

A rough estimate of the total energy of the membrane can be attained by multiplying the ground state energy E_0 per boson with the number of particles from equation (21). Using relation (18) we get:

$$E_m = E_0 N \simeq \frac{r_h}{8\pi^{3/2}}\left(\frac{f}{\beta}\right)^{\frac{1}{4}}\xi_0 = M\frac{\xi_0}{\sigma} \tag{135}$$

M is the gravitating mass of the holostar. The bosonic energy of the membrane is comparable to the gravitating mass of the holostar, giving further support to the holographic principle. However, E_m does not include the gravitational binding energy of the bosons within the membrane. The classical holostar equations predict a zero energy-density within the membrane, so that one expects that the binding energy is exactly opposite to E_m.

The argument can be turned around. From the holostar solution it is known that the tangential pressure of the membrane has a stress-energy "content" equal

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[44]However, it will not be possible to regard the membrane as a continuous surface, as has been done in the semi-classical approach in this paper.

to the holostar's gravitating mass. Under the assumption, that (i) the membrane consists of bosons, (ii) each boson in the membrane is in its ground state (with a wavelength roughly equal to the proper circumference of the holostar) and (iii) the mass energy within the membrane (excluding gravitational binding energy) can be estimated by simply summing up the individual boson energies, the total number of bosons constituting the membrane can be estimated via equations (134) and (135):

$$N_m \simeq M/E_0 \simeq \frac{\pi r_h^2}{\hbar} \frac{1}{\xi_0} = \frac{\sigma}{\xi_0} N \tag{136}$$

This estimate gives the same order of magnitude for the number of bosons in the membrane, N_m, and the number of particles within the holostar's interior, N.

14 The zero temperature case

In this section I discuss a "zero-temperature" holostar. I have put in this section rather for the completeness of coverage than being convinced that a zero-temperature holostar exists. However, the reader may judge for himself.

In the zero-temperature case the bosonic contribution to the (interior) mass-energy density and entropy can be neglected with respect to the fermions. At $T = 0$ the holostar's interior should be essentially free of bosons, all of which will have assembled in the membrane as the state of lowest energy. This will not be an option for the fermions, which are subject to the exclusion principle.

In the zero-temperature case the fermi-distribution in momentum space is given by a Heavyside step-function, which is unity for low momenta and falls off to zero abruptly at the fermi-momentum p_F. The fermi-momentum is nothing else than the chemical potential at $T = 0$.

The number of relativistic fermions in an interior spherical shell of volume δV can be calculated as follows:

$$\delta N = \frac{f}{2\pi^2 \hbar^3} \delta V \frac{p_F^3}{3} \tag{137}$$

where p_F is the Fermi-momentum.

The energy in the shell is given by:

$$\delta E = \frac{f}{2\pi^2 \hbar^3} \delta V \frac{p_F^4}{4} = \frac{3}{4} p_F \delta N \tag{138}$$

The mean energy of a highly relativistic fermion within the shell is lower than its fermi-momentum p_F, because all momenta up to p_F are occupied. In 3D-space the average momentum is $3/4\, p_F$.

The energy of the shell per proper volume must be equal to the mass-energy density of the holostar solution. Therefore:

$$\frac{\delta E}{\delta V} = \frac{f}{2\pi^2 \hbar^3} \frac{p_F^4}{4} = \frac{1}{8\pi r^2} \tag{139}$$

From this the fermi-energy p_F can be read off:

$$p_F{}^4 = \frac{\pi \hbar^3}{f} \frac{1}{r^2} \tag{140}$$

Inserting $p_F(r)$ from above into equation (137) and using equation (20) for the volume element we can determine the number of fermions within the shell:

$$\delta N = \frac{2}{3} \left(\frac{f}{\pi \beta} \right)^{\frac{1}{4}} \frac{r \delta r}{\hbar} = \frac{1}{3\pi} \left(\frac{f}{\pi \beta} \right)^{\frac{1}{4}} \delta S_{BH} \tag{141}$$

S_{BH} is the Bekenstein-Hawking entropy attributed to the shell.

According to the derivation in the last section we should now compare the thermodynamic entropy with the Hawking entropy in order to determine β. But there is a difficulty with this approach: The thermodynamic entropy of a zero-temperature holostar is zero. All states within the fermi-sphere are occupied. There is just one microscopic configuration for such a degenerate macroscopic state.

What seems to be possible, though, is to compare the momentum of the fermions at the holostar's surface with the Hawking temperature (at infinity). In order to do this, we have to establish a relation between the local "temperature" and the fermi-energy at the holostar's surface. Let us consider the process, where a thin shell of particles is added to the holostar. Any fermion in the newly added shell has an energy given by equation (138). The total energy of the shell is given by:

$$\delta E = \frac{3}{4} p_F \delta N \tag{142}$$

If any fermion carries an "intrinsic" entropy σ_0, the entropy of the newly added shell will be given by

$$\delta S = \sigma_0 \delta N \tag{143}$$

Combining this with the known thermodynamic relation $\delta S = \delta E / T$, we find:

$$p_F = \frac{4}{3} \sigma_0 T_0 \tag{144}$$

We can determine σ_0 by comparing the "temperature" T_0 at the holostar's surface with the Hawking temperature (both temperatures compared at infinity):

$$\sigma_0 = 3\pi \left(\frac{\pi \beta}{f} \right)^{\frac{1}{4}} \tag{145}$$

I haven't found a way to give accurate numerical figures for σ_0 in the zero temperature case, as was possible in the case of non-zero temperature. This would require us to know both f and β. Furthermore, one runs into severe problems, when one tries to interpret the "intrinsic" entropy σ_0 and the "temperature" T_0 (derived from the fermi momentum) in a thermodynamic sense. For instance, with $\mu_F = p_F$ the chemical potential per "temperature" naively is given by

$$u_F = \frac{p_F}{T_0} = \frac{4}{3}\sigma_0 \qquad (146)$$

One could now try to use the above relation to calculate the intrinsic entropy σ_0 from the thermodynamic equations of an ultra-relativistic gas of fermions at *non-zero* temperature, assuming that the "intrinsic" entropy per fermion is equal to the thermodynamic entropy given by equation (54). This would allow us to express σ_0 as a function of u_F. But this approach fails. The implicit equation for u_F has no solution. Furthermore, equation (146) is not even symmetric in u_F, so instead of getting two solutions u_F and $-u_F$, which can be interpreted as particle/anti-particle pair, we just get a nonsensical imaginary result. The failure of this approach is not quite unexpected. It doesn't really make sense to use the $T = 0$ Fermi-distribution for the calculation of energy- and number-densities and then set $T = p_F/u_F \neq 0$ in order to get rid of the undesired result $S = 0$ for the thermodynamic entropy. Quite obviously it requires a considerable amount of "creative cheating" in order to make a zero-temperature, zero entropy holostar compatible with the Hawking entropy and temperature relations.

An approach which quite likely is not correct either, but which at least leads to some sensible numerical figures, is to compare the entropy per fermion σ in the simple model of section 3 (a holostar, whose interior consists only out of fermions) with the "intrinsic entropy" σ_0 of a particle in the "zero-temperature" holostar, which is given by equation (145). The value of σ is given by equation (18). The ratio of both quantities turns out as:

$$\frac{\sigma_0}{\sigma} = \frac{3}{4\pi^{\frac{1}{4}}} \simeq 0.563 \qquad (147)$$

Knowing σ (for the non-zero temperature case), one might then be able to determine σ_0 via the above ratio. In the more sophisticated model of section 4 the entropy σ for a (non-zero) temperature holostar consisting only out of fermions has been shown to be $\sigma \simeq 3.38$, so that σ_0 might be estimated as

$$\sigma_0 \simeq 1.90$$

With σ_0 the total number of particles N can be calculated. It turns out larger than in the non-zero temperature case by roughly a factor of 1.8 (assuming $\sigma \simeq 3.38$ for the non-zero temperature case):

$$N = \frac{A}{4\sigma_0 \hbar} \approx 0.13 \frac{A}{\hbar} \qquad (148)$$

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Up to somewhat different constant factors the zero temperature model produces essentially the same results as the non-zero temperature model discussed in the previous sections. However, for $T = 0$ the thermodynamic entropy of the interior fermions is zero. The membrane doesn't contribute to the entropy anyway. Therefore a zero temperature holostar should have no appreciable thermodynamic entropy, which is in gross contradiction to the Hawking entropy-area law. Giving the fermions an "intrinsic" entropy can solve the problem, but not in a truly satisfactory way. Therefore I rate it doubtable that the the zero-temperature case is a physically acceptable description for a compact self gravitating object.

15 Rotation

In order to study collision or accretion processes involving the new type holostar solutions it will be necessary to find a solution that describes a rotating object.

Some properties of a yet to be found rotating axially-symmetric holostar solution might be inferred from the spherically symmetric solution. For a first approximation one could assume that the holostar rotates stiffly, at least for small rotation speeds. Unfortunately this requires infinitesimally small rotation rates $d\varphi/dt$ for a large holostar, in order that the holostar's surface doesn't rotate faster than the local velocity of light.

On the other hand, the event horizon of a Kerr black hole is known to rotate stiffly, irrespective of the rotation speed. Although there is no theorem like Birkhoff's theorem for axially-symmetric space-times, it is not unreasonable to assume that the exterior space-time of a rotating holostar is similar, if not identical, to the Kerr-metric. If this is true, at least the surface of a holostar should rotate stiffly. Furthermore one can expect that a rotating holostar will have no differential rotation within any interior spherical shell, although spherical shells with different radial coordinate positions[45] might rotate differentially with respect to each other. If there is differential interior rotation, it is quite probable that an interior observer will not be able to discern any peculiar angular motion due to differential rotation of the interior shells. The proper radial distance between adjacent spherical shells is huge, due to the r/r_0-dependence of the radial metric coefficient. Therefore, as long as the differential rotation doesn't depend exponentially on the radial coordinate value, an interior observer will not be able to notice the differential rotation of the shell with respect to an adjacent shell, even if the proper distance to the adjacent shell is equal to the Hubble-radius of the interior observer. This is due to the fact, that the radial coordinate distance between the shells becomes infinitesimally small for any fixed proper radial distance.

It is quite clear, that a holostar's rotation rate is limited. It must rotate less than the maximum rotation rate of a Kerr black hole: A maximally rotating (extreme) Kerr black hole is characterized by the property, that the proper

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[45]Keep in mind that for a rotating holostar it is not trivial to determine the geometric interpretation of a chosen "radial coordinate".

velocity of the (stiffly rotating) event horizon equals the velocity of light. For the holostar solution this would imply that the rotation speed of the membrane, which lies slightly outside the gravitational radius, would exceed the velocity of light.

15.1 A bound for the mean spin-alignment of the interior particles

This upper bound enables us to get some valuable information with respect to the interior structure of a rotating holostar. For this purpose it is instructive to reflect on how the rotation will manifest itself locally in the holostar's interior. The overall rotation is expected to force the interior particles to align their spins and orbital momenta along the rotation axis. I assume, that the rotation doesn't change the number of interior particles.[46] In this case the entropy (and surface area) should remain constant. The maximum rotation will be achieved, when the spins and orbital momenta of all interior particles are aligned. The dominant particle species within a holostar, at least with respect to the number of particles, are ultra-relativistic particles. If we neglect the orbital momenta, we can determine the fraction of the total angular momentum of a holostar, J_H, due to the alignment of its spins:

$$J_H = \overline{j}\hbar N = \frac{\overline{j}}{4\sigma} A \tag{149}$$

$\overline{j}$ is the expectation value of the spin quantum number of the ultra-relativistic particles in direction of the exterior rotation axis. For a maximally aligned holostar (meaning the j_z component of the spins of all particles point into the direction of the exterior rotation axis) $\overline{j}$ will be equal to the sum of the spin-quantum numbers of all the particles composing the holostar divided by the total number of particles. For a large holostar approaching the size of the universe, the dominant interior (relativistic) particle species will be the neutrinos with $j_F = 1/2$ and possibly photons with $j_B = 1$.

If orbital angular momentum is included, the total angular momentum of the holostar will be higher than the value in equation (149). Alternatively $\overline{j}$ can be interpreted as the expectation value of the spin of the particles in direction of the rotation axis, including the mean orbital momentum of the particles (if there is any). To get an estimate of the maximum possible rotation rate of a holostar we can compare the angular momentum of a maximally aligned holostar to that of a maximally rotating (extreme) Kerr black hole.

For an extreme Kerr-Black hole there is a definite relationship between its angular momentum, J_K, and surface area:

$$J_K = \frac{A}{8\pi}$$

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[46]This requires that energy must be transferred adiabatically to the holostar in order to spin it up from zero to maximum angular momentum.

The holostar's angular momentum can never exceed the angular momentum of an extreme Kerr-black hole, which requires $J_H < J_K$. Comparing the two angular momenta we find:[47]

$$\frac{J_H}{J_K} = 2\overline{j}\frac{\pi}{\sigma} < 1$$

This can be interpreted as a bound on the mean spin quantum number of the interior particles of a rotating holostar:

$$\overline{j} < \frac{1}{2}\frac{\sigma}{\pi} \approx \frac{1}{2} \cdot (1.004\ldots1.07) \tag{150}$$

In our simple thermodynamic model the value of σ, the entropy per fermion, is very close to π. The exact value of σ depends on the ratio between the bosonic and the fermionic degrees of freedom and on how the chemical potentials of bosons and fermions are related. For all reasonable values of f_F and f_B, the ratio σ/π is larger than 1 although only by a small percentage, as can be seen from the tables in section 8.

For the realistic thermodynamic model, i.e. when the chemical potential of the bosons is zero (Table 1) and for realistic values of $f_B/f_F < 3000$, σ/π attains its maximum for $f_B/f_F = 0$ and its minimum at roughly $f_B/f_F = 8$:

$(\sigma/\pi)_{max} \approx 1.076$ for $f_B = 0$
$(\sigma/\pi)_{min} \approx 1.004$ for $f_B/f_F \approx 8$

For the "abnormal supersymmetric phase", i.e. when the chemical potential of the bosons is opposite in sign to the chemical potential of the fermions (Table 2), we find:

$(\sigma/\pi)_{min} \approx 1.067$ for $f_B = 0$
(σ/π) ≈ 1.073 for $f_B = f_F$

In any case, whatever the combination of f_F and f_B and the relation between the chemical potentials might be, one can come very close to the angular momentum of a maximally rotating Kerr-Black hole, by simply aligning the spins of the ultra-relativistic fermions (and bosons) of the spherically symmetric holostar solution. If the (mean effective) spin quantum number of the ultra-relativistic particles within the holostar is larger than $1/2$, it is possible to exceed the angular momentum of a maximally rotating Kerr-black hole, simply by aligning a large proportion of the spins, as can be seen from equation (150). If the interior particles have an appreciable orbital angular momentum, which was neglected in the determination of J_H, the Kerr-limit will be exceeded, even if all of the ultra-relativistic particles are spin-$1/2$ particles (i.e. particles with the lowest non-zero spin quantum possible). This is physically unacceptable. The outer regions of the holostar would rotate with a velocity larger than the speed of light.

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[47]Note that both objects are compared assuming that their surface areas are equal. This seems the most natural assumption. A rotating holostar, whose number of particles is equal to the non-rotating case, should have the same entropy and thus the same surface area. We therefore should compare the rotating holostar with a Kerr black hole of the same area.

15.1.1 interior particle spin $j = 1/2$

Under the assumption, that the holostar consists only out of spin 1/2 fermions, aligning the spins of all particles already yields a total angular momentum that is quite close to the maximum angular momentum possible, J_K:

$$J_H = \frac{\pi}{\sigma} J_K = 0.9297 J_K \qquad (151)$$

$J \approx 0.9\, J_K$ is quite close to the rotation rates expected for black holes formed from realistic gravitational collapse or accretion processes, when the angular momentum of the collapsing matter is taken into account. In fact, very recently the mass and angular momentum of the black hole in the center of our galaxy has been measured by analyzing the frequency spectrum of X-ray flares coming from the galactic center [1]. The measurements allow a very precise determination of the angular momentum variable $a = J/J_K$ (a denotes the ratio of the angular momentum of a black hole to its maximum possible value, which is given by the angular momentum of an extreme Kerr black hole). The data allow two solutions, characterized by a low ($M = 2.79(4) \cdot 10^6 M_\odot$) and a high ($M = 4.75(7) \cdot 10^6 M_\odot$) gravitational mass of the black hole. Although the masses differ by a factor of almost two, the angular momentum parameters are nearly identical, $a = 0.9937(7)$ for the low and $a = 0.991(2)$ for the high mass solution. Both values are very close to the prediction $a = 0.9297$ for a maximally aligned fermionic holostar (Eq. (151)).

The argument can be turned around. There appears to be no basic physical law that forbids the interior particles of a holostar to align their spins along a common axis. If this is so, the interior (massless) particles of a rotating holostar should be mostly spin 1/2 particles or a mixture of spin 0, spin 1/2 and spin 1 particles, with not too high a contribution of particles with higher spin, otherwise the Kerr-limit would be exceeded.[48] Furthermore, the interior particles of a rapidly rotating holostar cannot have high orbital angular momenta, otherwise the Kerr-limit would be exceeded even if the holostar consisted exclusively out of spin-1/2 particles.

15.1.2 interior particle spin $j = 1/2$ and $j = 1$

If the holostar contains one massless fermionic and one massless bosonic species, such as the neutrino and the photon, it not possible to align all particles. Let us construct a very simple example. Consider the case $f_F = f_B$. Let all of the fermions have spin $\overline{j_F} = 1/2$ and all of the bosons spin $\overline{j_B} = 1$. According to the formula and tables given in section 8, the number density of the fermions + anti-fermions per proper volume is a factor of $w = (w_N + \overline{w_N})/2 \simeq 1.7877$ higher than the number density of the bosons.

If the spins of all particles are aligned, the mean expectation value of all spins in the direction of the alignment-axis is given by:

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[48]This is quite similar to the situation in loop quantum gravity, where the number of states of the area operator (within a given small area range) is dominated by spin 1/2 links for large areas.

$$\overline{j} = \frac{w}{1+w}\overline{j_F} + \frac{1}{1+w}\overline{j_B} = 0.6794 \tag{152}$$

This is larger than the bound of equation (150), evaluated for $\sigma = 3.2299$:

$$\overline{j} < \frac{1}{2}\frac{\sigma}{\pi} = 0.5141 \tag{153}$$

This bound cannot be exceeded, therefore either it is impossible to align all of the spins or there must be a significant fraction of spin-0 bosons. If only the fermions are aligned, we get:

$$\overline{j} = \frac{w}{1+w}\frac{1}{2} = 0.3206 \tag{154}$$

so that angular momentum of the holostar due to the alignment of all of its spin-1/2 particles is roughly 62% of the maximum spin.

It seems awkward to postulate, that the fermionic spins can be aligned along a common axis and the bosonic spins not. If one aligns all spins, there is no problem when the holostar consists only out of fermions, whereas for equal fermion and boson numbers the spin-limit is exceeded (with the assumption that all fermions have spin-1/2 and all bosons spin-1). If one knows how the spins are distributed among the different particle-species, such as in certain supersymmetric models, it is possible to check whether aligning all of the spins violates the spin-limit $j_{max} = \sigma/(2\pi)$. With the simplified assumption that all bosons have spin-1 and all fermions spin-1/2, we find that there must be at least 10.45 fermionic degrees of freedom per bosonic degree of freedom, in order that the spin-limit is not exceeded.

15.2 supersymmetric models

An interesting value is the spin-limit for equal bosonic and fermionic degrees of freedom. According to equation (153) its value is given by $j_{ss} = 0.5141$. One might use this value in order to restrict the various supersymmetric models. For example, $N = 8$ supersymmetry has a mean-spin of the fermions of $\overline{j_F} = 5/8$ and a mean spin of the bosons $\overline{j_B} = 15/32$. Using equation (152) and replacing $\overline{j_F} \to 5/8$ and $\overline{j_B} \to 15/32$ leads to $\overline{j} = 0.569$, which exceeds the limit, whereas $N = 4$ supersymmetry leads to $\overline{j} = 0.410$, which is within the limit.

In any case, the spin-limit provides quite a stringent constraint, which can be violated quite easily, even without taking the angular momentum of the particle's motion into account. Therefore we are led to the assumption, that a rotating holostar most likely acquires its angular momentum predominantly due to the alignment of its interior particles. There should be neither an appreciable contribution from the higher spin particles, such as spin-1 bosons, nor a significant angular momentum contribution from the interior particles. Note also, that if the angular momentum value of a large charged holostar can be interpreted as the alignment of its charged spin 1/2 particles along a common axis, this might be an "explanation" for the g-factor of two for a rotating charged Kerr-Newman black hole.

15.3   the zero temperature case

How does a zero-temperature holostar, as discussed in section 14 fit into this picture? For a zero temperature holostar one finds a numerically different result for the maximum spin expectation value:

$$\frac{J_H}{J_K} = 2\bar{j}\frac{\pi}{\sigma_0} \simeq 3.32\bar{j} < 1$$

or

$$\bar{j} \approx < \frac{1}{3}$$

If more than roughly two thirds of the spin 1/2 fermions are aligned along a particular axis, the angular momentum of the zero-temperature holostar will exceed the angular momentum of the extreme Kerr-solution. If the interior particles have higher spin or orbital angular momentum, such a situation will occur at even lower relative alignments. Rotation rates exceeding the extreme Kerr-rate are physically not acceptable. On the other hand, from the viewpoint of the interior observer there seems to be no good reason, why not all of the spin's can be aligned along a common axis. Therefore the above finding might be interpreted as further evidence, that a zero-temperature black hole is not physically realized.

15.4   local CP-violation

Quite interestingly, a rotating holostar could be a natural cause for an (apparent) C and/or P violation. As has been shown in [7] the motion of any "ordinary" particle in the holostar becomes nearly radial and outward directed, whenever the particles can be considered as non-interacting. Such a non-interaction condition is evidently satisfied by the neutrinos in the outer regions of a holostar of the size of the universe. If the spins of the radially outward moving neutrinos are forced to line up with respect to a given exterior rotation axis, the holostar will be divided into two half-spheres with a distinct matter-antimatter asymmetry (if the neutrinos have just one definite helicity state as assumed by the Standard Model). Under the alignment-constraint imposed by the exterior rotation axis the antineutrinos, with spin in direction of flight, will preferentially move in the direction of the external rotation axis, whereas the neutrinos with opposite helicity will preferentially move in the opposite direction. If the holostar attains its maximum rotation rate, i.e. all of the fermion spins are aligned with respect to the rotation axis, there will be an almost perfect neutrino/antineutrino asymmetry between the two half-spheres. Neutrinos moving radially outward will dominate one half-sphere, anti-neutrinos the other.

16 Discussion and Outlook

A simple thermodynamic model for the holostar solution has been presented which fits well into the established theory of black holes. From the viewpoint of an exterior observer the holostar appears very similar to a classical black hole. The modifications are minor and only "visible" at close distance: The event horizon is replaced by a two dimensional membrane with high tangential pressure, situated roughly a two Planck coordinate lengths outside of the gravitational radius. The pressure of the membrane is equal to the pressure derived from the so called "membrane paradigm" for black holes [10], which guarantees, that the holostar's action on the exterior space-time is indistinguishable from that of a same-sized black hole. The membrane has zero energy-density, as expected from string theory. The interior matter is singularity free and can be interpreted as a radial collection of classical strings, attached to the holostar's spherical boundary membrane. Each string segment attached to the membrane occupies a membrane segment of exactly one Planck area. The string tension falls with radius and is inverse proportional to the string length, as measured by an asymptotic observer at spatial infinity.

The interior matter state can be interpreted in terms of particles. In this paper a very simple thermodynamic model of an ideal ultra-relativistic gas was discussed. Temperature and entropy of the holostar are of microscopic origin and exactly proportional to the Hawking temperature and -entropy. The number of interior particles within the holostar is proportional to the proper area of the boundary-membrane, measured in Planck units, indicating that the holographic principle is valid for compact self-gravitating objects of arbitrary size.

The surface temperature of a holostar measured at infinity is proportional to the Hawking temperature. By this correspondence one can set up a specific relation between the Hawking temperature (measured at infinity), the interior radiation temperature and the interior matter density. This correspondence allows an experimental verification of the Hawking-temperature law from the holostar's interior. By comparing the CMBR-temperature to the total matter density of the universe the Hawking temperature law has been experimentally verified to an accuracy of roughly 1 %. However, the numerical verification depends on the formula given in [5] for β (Eq. 78), which stills lacks a formal derivation.

The holostar solution has no singularity and no event horizon. Information is not lost: The total information content of the space-time is encoded in its constituent matter, which can consist out of strings or particles. Unitary evolution of particles is possible throughout the full space-time manifold. Every ultra-relativistic particle carries a definite entropy, which can be calculated when the number of ultra-relativistic fermionic and bosonic degrees of freedom is known.

The thermodynamics of the holostar solution indicate, that chemical potentials play an important role in the stabilization of self gravitating systems. The holostar requires a non-zero chemical potential, proportional to the local radiation temperature, of at least one fermionic ultra-relativistic species. Different thermodynamic models can be constructed, which are characterized by the ratio

of bosonic to fermionic degrees of freedom.

The non-zero chemical potential of the ultra-relativistic fermions acts as a natural cause for a significant matter-antimatter asymmetry in a spherically symmetric curved space-time. For any ultra-relativistic fermionic particle there are never more than 0.09 anti-particles, meaning the total fraction of anti-fermions is always less than 8 % of the total number of particles and anti-particles of a given species. Therefore at least 84 % of the original energy-density of an ultra-relativistic fermion gas "survives" in ordinary fermionic matter, if the temperature falls below the mass-threshold and less than 8 % anti-fermions annihilate with their matter counterparts.

Although the interior structure of the non-rotating holostar can be considered as fairly well understood, not much is known about the membrane and the astrophysically interesting case of a rotating holostar. These two topics present themselves as a very interesting themes of future research. Some arguments with respect to the interior structure of the membrane and the properties of a - yet to be found - rotating holostar solution were given. It has been proposed, that the membrane consists of a gas of bosons at a temperature far below the Bose-temperature of the membrane, forming a single macroscopic quantum state. Evidence has been presented that a rotating holostar will acquire its angular momentum preferentially by the alignment of its interior particles along the rotation axis. Whether these tentative "predictions" point into the right direction can only be answered by future research.

Having two or more solutions for the field equations (black hole vs. holostar) makes the question of how these solutions can be distinguished from each other experimentally an imminently important question. Can we find out by experiment or observation, which of the known solutions, if any, is realized in nature? At the present time the best argument in favor of the holostar solution appears to be the accurate measurement of the Hawking temperature via the CMBR-temperature and the matter-density of the universe.

Yet it would be helpful if more direct experimental evidence were available. Due to Birkhoff's theorem the holostar cannot be distinguished from a Schwarzschild black hole by measurements of its exterior gravitational field. But whenever holostars come close to each other or collide, their characteristic interior structure should produce observable effects, which deviate from the collisions of black holes. Presumably a collision of two holostars will be accompanied by an intense exchange of particles, with the possible production of particle jets along the angular momentum axis.

In accretion processes the membrane might produce a noticeable effect. The rather stiff membrane with its high surface pressure might be a better "reflector" for the incoming particles, than the vacuum-region of the event horizon of a Schwarzschild-type black hole. There are observations of burst-like emissions from compact objects, which are assumed to be black holes because of their high mass ($M > 3 - 5M_\odot$), but that have spectra rather characteristic for neutron stars, i.e. of particles "hitting" a stiff surface. A more accurate observation of these objects might provide important experimental clues to decide the issue.

A high density of bosons in the membrane, with a mean separation compa-

rable to the Planck-length, might induce copious particle-interchange reactions, similar to what is expected when particles cross a so called domain-wall. For holostars of sub-stellar size ($r_h \approx 1\,km$) the local temperature at the membrane becomes comparable to the nucleon rest mass energy. A rather hot particle gas at the position of the membrane could produce noticeable effects with respect to the relative abundances of the "reflected" particles, due to high energy interactions with the constituent particles of the membrane or the holostar's interior.

On the other hand, the extreme surface red-shifts on the order of $z \approx 10^{20}$ for a solar mass holostar, and larger yet for higher mass objects ($z \propto \sqrt{M}$), might not allow a conclusive interpretation of the experimental data with regard to the true nature of any such black hole type object.

The most promising route therefore appears to be, to study the holostar from its interior. In [7] it has been demonstrated, that the holostar has the potential to serve as an alternative model for the universe. The recent WMAP-measurements have determined the product of the Hubble constant H times the age of the universe τ to be $H\,\tau \simeq 1.02$ experimentally with $H = 71\,(km/s)/Mpc$ and $\tau = 13.7\,Gy$. The holostar solution predicts $H\,\tau = 1$ exactly. There are other predictions which fit astoundingly well with the observational data. This in itself is remarkable, because the holostar-solution has practically no free parameters. It's unique properties arise from a delicate cancelation of terms in the Einstein field equations, which only occurs for the "special" matter density $\rho = 1/(8\pi r^2)$, leading to the "special" radial metric coefficient $g_{rr} = r/r_0$ (see [6] for the derivation). That the holostar solution with its completely "rigid" structure has so much in common with the universe as we see it today, either is the greatest coincidence imaginable, or not a coincidence at all.

With the holostar solution we have a beautifully simple model for a singularity free compact self gravitating object, which is easily falsifiable. Its metric and fields are simple, its properties are not. It is an elegant solution, as anyone studying its properties will soon come to realize. However, in science it is experiments and observations, not aesthetics, that will have to decide, which solution of the field equations has been chosen by nature. It is our task, to find out. The work has just begun.

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